

2.7 Residuals

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS residual 残差 • residual plot 残差图

Objective

Build the skills to answer exam questions on **residuals**.

You must be able to:

- calculate a **residual** 残差 as $y - \hat{y}$ (observed minus predicted)
- construct and interpret a **residual plot** 残差图
- use a residual plot to judge whether a **linear model** is appropriate

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Residuals

$$\text{residual} = y - \hat{y} = \text{observed} - \text{predicted}.$$

A **positive** residual means the point lies **above** the line.

■ Residual plots

Plot the residuals against x (or $\hat{y}$):

- **random scatter** with no pattern $\rightarrow$ the linear model is appropriate;
- a **clear pattern** (e.g. a curve) $\rightarrow$ a linear model is **not** appropriate.

2 Practice

2.1 Write the formula for a residual. [1]

2.2 An observed value is 18 and the model predicts 15. Find the residual. [2]

2.3 State what a residual plot with a clear pattern suggests. [1]

3 Exam-style questions

3.1 A residual equals [1]

- A $\hat{y} - y$
- B $y - \hat{y}$
- C $y + \hat{y}$
- D $y \times \hat{y}$

3.2 A residual plot showing random scatter suggests the linear model is [1]

- A inappropriate
- B appropriate
- C curved
- D wrong

3.3 A prediction is $\hat{y} = 40$, but the actual value is $y = 46$.

(a) Find the residual. [2]

(b) State whether the point is above or below the line. [1]

Solutions

2.1 residual = $y - \hat{y}$.

2.2 $18 - 15 = 3$.

2.3 a linear model is not appropriate for the data.

3.1 B.

3.2 B.

3.3 (a) $46 - 40 = 6$. (b) above the line (positive residual).