Past-paper walk-through

Carrying Out a Test for the Slope of a Regression Model ▪ 9.5

eXercise

Questions in this set

- 2023 Q5

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

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Wildlife biologists are interested in the health of tule elk, a species of deer found in California. An important measurement of tule elk health is their weight. The weight of a tule elk is difficult to measure in the wild. However, chest circumference, which is believed to be related to the weight of a tule elk, can easily be measured from a safe distance using a harmless laser. A study was done to investigate whether chest circumference, in centimeters (cm), could be used to accurately estimate the weight, in kilograms (kg), of male tule elk. For the study, wildlife biologists captured 30 male tule elk, measured their chest circumference and weight, and then released the elk. The data for the 30 male tule elk are shown in the scatterplot.

[Scatterplot titled with axes "Chest Circumference (cm)" (x-axis, 110 to 160) and "Weight (kg)" (y-axis, 50 to 250). The points show a strong, positive, roughly linear association: e.g. points near (113,90), (120,93), (121,98), (125,103), (130,145),

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(133,148), (135,150), (137,155), (138,160), (140,163), (141,175), (142,178), (143,180), (144,183), (145,204), (146,207), (147,210), (148,213), (149,233), (150,236), (152,238), (153,240), (155,241), continuing up to about (156,241), with the cloud of points trending upward from lower-left to upper-right.]

(a) Describe the relationship between chest circumference and weight of male tule elk in context.

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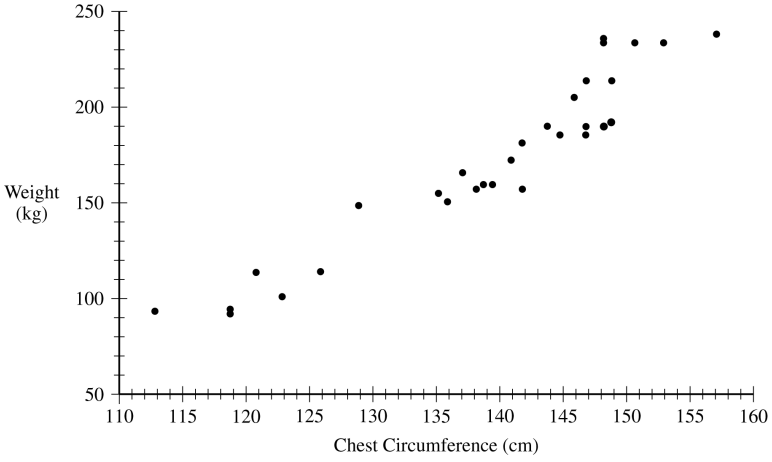
250
200
150
100
50
Weight
(kg)
110
115
120
125
130
135
140
145
150
155
160
Chest Circumference (cm)

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(b)(i) Following is the equation of the least-squares regression line relating chest circumference and weight for male tule elk.

$$\text{predicted weight} = -350.3 + 3.7455(\text{chest circumference})$$

The weight of one male tule elk with a chest circumference of 145.9 cm is 204.3 kg. Using the equation of the least-squares regression line, calculate the predicted weight for this male tule elk. Show your work.

(b)(ii) Calculate the residual for this male tule elk. Show your work.

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(c) The equation of the least-squares regression line relating chest circumference and weight for male tule elk is repeated here.

$$\text{predicted weight} = -350.3 + 3.7455(\text{chest circumference})$$

Interpret the slope of the least-squares regression line in context.

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Wildlife biologists are interested in the health of tule elk, a species of deer found in California. An important measurement of tule elk health is their weight. The weight of a tule elk is difficult to measure in the wild. However, chest circumference, which is believed to be related to the weight of a tule elk, can easily be measured from a safe distance using a harmless laser. A study was done to investigate whether chest circumference, in centimeters (cm), could be used to accurately estimate the weight, in kilograms (kg), of male tule elk. For the study, wildlife biologists captured 30 male tule elk, measured their chest circumference and weight, and then released the elk. The data for the 30 male tule elk are shown in the scatterplot.
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(d)(i) The sambar, another species of deer, is similar in size to the tule elk. The slope of the population regression line relating chest circumference and weight for all male sambars is 4.5 kilograms per centimeter. A wildlife biologist wants to determine whether the slope of the population regression line for male tule elk is different than that for male sambars. Let β represent the slope of the population regression line for male tule elk. The wildlife biologist conducted a test of the following hypotheses using the sample of 30 tule elk.

$$H_0 : \beta = 4.5$$

$$H_a : \beta \neq 4.5$$

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The test statistic was calculated to be 3.408. Assume all conditions for inference were met.

Determine the *p*-value of the test.

(d)(ii) At a significance level of $\alpha = 0.05$, what conclusion should the wildlife biologist make regarding the slope of the population regression line for male tule elk? Justify your response.

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(a) Mark scheme

- The scatterplot reveals a strong, positive, roughly linear association between chest circumference and weight of male tule elk. There are no points that seriously deviate from the straight-line pattern of the points in the plot. Full credit needs a description that includes at least 3 of components 1-4 AND component 5: (1) direction of association (positive/increasing), (2) strength of association (strong), (3) form of association (linear or approximately linear), (4) unusual features (no points with large discrepancies from the straight-line pattern exhibited by most points - it is sufficient to simply state there are no unusual features/outliers), (5) context - association between chest circumference and weight of tule elk (minimally sufficient to refer to the association between chest

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circumference/measurement and weight without explicitly naming tule elk). Partial credit if the response satisfies only 1-2 of components 1-4 AND component 5, OR at least 2 of components 1-4 but not component 5.

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(b)(i) Mark scheme

- Using the least-squares regression equation predicted weight = -350.3 + 3.7455(chest circumference), the predicted weight of a male tule elk with a chest circumference of 145.9 cm is -350.3 + 3.7455(145.9) $\approx$ 196.17 kg. NOTE: parts (b)(i) and (b)(ii) are scored TOGETHER as ONE holistic unit in the official scoring guidelines, so this leaf carries this part's full mark. Full credit for the combined part (b) needs both: (1) provides the correct predicted value 196.17 kg in part (b)(i) with work shown, (2) provides the correct residual 8.13 kg in part (b)(ii) [graded together with leaf 5(b)(ii)], or a value consistent with the predicted weight calculated here, with work shown. Partial credit if the response satisfies only one of the two components, OR gives both correct values with no work

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shown. A response using an undefined or incorrect variable to identify the work shown (e.g. x or p-hat instead of predicted weight) does not satisfy component 1. An arithmetic or transcription error can be ignored if correct work is shown.

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(b)(ii) Mark scheme

- The residual for the male tule elk with chest circumference 145.9 cm and actual weight 204.3 kg is residual = actual − predicted = $204.3 - 196.17 \approx 8.13$ kg. This leaf's credit is bundled with 5(b)(i) in the official scoring guidelines (Part (b) of the SG scores both sub-parts together as one holistic unit), so it carries 0 of this question's marks on its own - but a correct residual of 8.13 kg (or a value consistent with the predicted weight from part (b)(i)), with work shown, is still expected to complete credit for part (b). An arithmetic or transcription error can be ignored if correct work is shown.

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(c) Mark scheme

- The value of the slope of the least-squares regression line is 3.7455 (may be rounded to 3.746, 3.75, or 3.7, but not to 3 or 4). This value indicates that the predicted weight of a tule elk increases by 3.7455 kilograms for each additional centimeter of chest circumference. Full credit needs all 3 components: (1) identifies the value of the slope as 3.7455 (a response that only contains 3.7455 in the interpretation still satisfies this), (2) provides an interpretation that references an increase of a number of kilograms of weight for each one-centimeter increase in chest circumference (units must be correctly specified for both weight and length), (3) indicates the slope represents a change in a PREDICTION using non-deterministic language such as 'predicted,' 'estimated,' 'expected,' or

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'average' (not a deterministic claim that weight definitely increases). Partial credit if only 2 of the 3 components are satisfied. It is not required to refer specifically to the 'least-squares regression line.'

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(d)(i) Mark scheme

- The degrees of freedom for the test of the population slope (relating chest circumference and weight for male tule elk, testing H0: β=4.5 against a two-sided alternative, versus the sambar species' known slope of 4.5 kg/cm) are n $-$ 2 = 30 $-$ 2 = 28. The t-table shows that for 28 degrees of freedom, the p-value for a one-sided test would be 0.001. Because this is a two-sided test, the p-value is (2)(0.001) = 0.002. NOTE: parts (d)(i) and (d)(ii) are scored TOGETHER as ONE holistic unit in the official scoring guidelines, so this leaf carries this part's full mark. Full credit for the combined part (d) needs all 3: (1) gives a correct p-value of 0.002, consistent with the two-sided alternative hypothesis, (2) provides a correct comparison of the p-value to alpha AND a correct decision about the

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null/alternative hypothesis [graded together with leaf 5(d)(ii)], (3) provides a conclusion statement in context, consistent with and in terms of the alternative hypothesis, using non-deterministic language [also from part (d)(ii)]. Partial credit if only 2 of the 3 components are satisfied.

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(d)(ii) Mark scheme

- Because the p-value (0.002) is less than $\alpha = 0.05$, reject the null hypothesis. There is sufficient statistical evidence that the population slope for the linear regression of weight vs. chest circumference for male tule elk is different from 4.5 kg/cm (i.e. different from the population slope for male sambars). This leaf's credit is bundled with 5(d)(i) in the official scoring guidelines (Part (d) of the SG scores both sub-parts together as one holistic unit), so it carries 0 of this question's marks on its own - but a correct decision (reject H0) with a correctly-contextualized, non-deterministic conclusion is still expected to complete credit for part (d). An explicit decision statement is not strictly required if the decision is clearly implied within the conclusion.