

Exercise walk-through

Carrying Out a Chi-Square Test for Homogeneity or Independence ■ 8.6

eXercise

You must be able to

- compute χ^2 from a two-way table and find the **p-value**
- compare it to α and reach a decision
- state a conclusion about association in context

Method — The procedure

- 1 State the hypotheses; check conditions (expected counts ≥ 5).
- 2 Compute the expected counts, then $\chi^2 = \sum \frac{(O - E)^2}{E}$, with $df = (r - 1)(c - 1)$.
- 3 Find the **p-value**.
- 4 Compare to α : a **small** p-value $\rightarrow$ **reject** $H_0 \rightarrow$ the variables are **associated**.

Method — Finding an expected count

Each expected count is $\frac{\text{row total} \times \text{column total}}{\text{grand total}}$. For a cell whose row totals 40 and column totals 50, with grand total 200:

$$E = \frac{40 \times 50}{200} = 10.$$

Compute every E , then sum $\frac{(O - E)^2}{E}$.

Practice 2.1 ■ 2 marks

State the steps to carry out a chi-square test for a two-way table.

Solution 2.1

Worked steps

state hypotheses and check conditions; compute expected counts and χ^2 ; find the p-value; compare to α and conclude **B1** **B1** *for two steps; for the full procedure.*

Practice 2.2 ■ 1 mark

State what a small p-value indicates about the two variables.

Solution 2.2

Method: The procedure

Worked steps

the two variables are associated (not independent) **B1**.

Practice 2.3 ■ 1 mark

A test of independence gives a p-value of 0.02 at $\alpha = 0.05$. State the decision.

Solution 2.3

Worked steps

reject H_0 ($0.02 < 0.05$) **B1**.

Exam-style 3.1 ■ 1 mark

A chi-square test of independence with a small p-value suggests the variables are

A independent

B associated

C exactly equal

D unrelated

Solution 3.1

Method: The procedure

A independent

B associated 

C exactly equal

D unrelated

Worked steps

B.

Exam-style 3.2 ■ 1 mark

A chi-square test rejects H_0 when the p-value is

- A** above α
- B** below α
- C** exactly zero
- D** exactly one

Solution 3.2

- A above α
- B below α
- C exactly zero
- D exactly one



Worked steps

B.

Exam-style 3.3 ■ 1 mark

A test of independence gives $\chi^2 = 10$, $df = 2$, p-value 0.007 at $\alpha = 0.05$.

- (a) State the decision.
- (b) State whether the variables are associated.
- (c) State the conclusion in context.

Solution 3.3

Method: The procedure

Worked steps

(a) reject H_0 **B1**. (b) yes, they are associated **B1**. (c) there is convincing evidence of an association between the two variables **B1**.