

Exercise walk-through

Setting Up a Chi-Square Test for Homogeneity or Independence ■ 8.5

eXercise

You must be able to

- distinguish a test of **homogeneity** 同质性 from a test of **independence** 独立性
- calculate the degrees of freedom $(r - 1)(c - 1)$

Method — Homogeneity vs independence

- **Homogeneity** — compares the distribution of one categorical variable across **several groups** (several samples).
- **Independence** — tests whether **two** categorical variables are associated within **one** population (one sample).

Both use the same statistic $\chi^2 = \sum \frac{(O - E)^2}{E}$ and the same degrees of freedom:

$$\text{df} = (r - 1)(c - 1).$$

Method — Which one, and its df

A 3×4 table (3 rows, 4 columns) has $df = (3 - 1)(4 - 1) = 6$. If the rows are **separate samples** (say, three schools), it is a test of **homogeneity**; if the table comes from **one sample** cross-classified by two variables, it is a test of **independence**.

Practice 2.1 ■ 2 marks

State the difference between a test of homogeneity and a test of independence.

Solution 2.1

Method: Which one, and its df

Worked steps

homogeneity compares one variable's distribution across several groups; independence tests whether two variables are associated in one population **B1** **B1**.

Practice 2.2 ■ 1 mark

Write the degrees of freedom for an $r \times c$ table.

Solution 2.2

Worked steps

$$(r-1)(c-1) \text{ B1.}$$

Practice 2.3 ■ 2 marks

For a 3×4 table, find the degrees of freedom.

Solution 2.3

Worked steps

$$(3 - 1)(4 - 1) = 2 \times 3 = 6 \quad \text{M1} \quad \text{A1}.$$

Exam-style 3.1 ■ 1 mark

The degrees of freedom for a two-way chi-square with r rows and c columns is

A rc

B $(r-1)(c-1)$

C $r+c$

D $rc-1$

Solution 3.1

A rc

B $(r-1)(c-1)$



C $r+c$

D $rc-1$

Worked steps

B.

Exam-style 3.2 ■ 1 mark

A test of whether two categorical variables are associated in one population is a test of

A homogeneity

B independence

C goodness of fit

D means

Solution 3.2

Method: Homogeneity vs independence

A homogeneity

B independence 

C goodness of fit

D means

Worked steps

B.

Exam-style 3.3 ■ 2 marks

A 2×3 table is analysed.

- (a) Find the degrees of freedom.
- (b) Name the test that compares distributions across several groups.

Solution 3.3

Method: Homogeneity vs independence

Worked steps

(a) $(2 - 1)(3 - 1) = 2$ **M1** **A1**. (b) a test of homogeneity **B1**.