

Exercise walk-through

Carrying Out a Chi-Square Test for Goodness of Fit ■ 8.3

eXercise

You must be able to

- compute $\chi^2 = \sum \frac{(O - E)^2}{E}$ with degrees of freedom = $k - 1$
- find the **p-value** and reach a decision
- state a conclusion in context

Method — The procedure

- 1 State the hypotheses and check conditions (all expected counts ≥ 5).
- 2 Compute $\chi^2 = \sum \frac{(O - E)^2}{E}$, with $df = k - 1$ (categories minus 1).
- 3 Find the **p-value** from the chi-square distribution.
- 4 Compare to α and conclude **in context**.

A **large** χ^2 gives a **small** p-value — stronger evidence against the claimed distribution.

Method — A worked statistic

For observed $(18, 12, 30)$ and expected $(20, 20, 20)$,

$$\chi^2 = \frac{(18 - 20)^2}{20} + \frac{(12 - 20)^2}{20} + \frac{(30 - 20)^2}{20} = 0.2 + 3.2 + 5.0 = 8.4,$$

with $df = 3 - 1 = 2$. This gives a p-value $\approx 0.015 < 0.05$ – reject the claimed distribution.

Practice 2.1 ■ 1 mark

State the degrees of freedom for a goodness-of-fit test with k categories.

Solution 2.1

Worked steps

$$k - 1 \text{ (B1)}.$$

Practice 2.2 ■ 1 mark

State what a large chi-square value suggests.

Solution 2.2

Method: The procedure

Worked steps

a large difference between observed and expected counts (evidence against H_0) **B1**.

Practice 2.3 ■ 1 mark

A goodness-of-fit test gives a p-value of 0.03 at $\alpha = 0.05$. State the decision.

Solution 2.3

Method: The procedure

Worked steps

reject H_0 ($0.03 < 0.05$) **B1**.

Exam-style 3.1 ■ 1 mark

The degrees of freedom for a goodness-of-fit test with 6 categories is

A 6

B 5

C 4

D 1

Solution 3.1

A 6

B 5



C 4

D 1

Worked steps

B.

Exam-style 3.2 ■ 1 mark

A large chi-square statistic gives a _____ p-value.

A large

B small

C zero

D negative

Solution 3.2

Method: The procedure

A large

B small



C zero

D negative

Worked steps

B.

Exam-style 3.3 ■ 1 mark

A goodness-of-fit test of a die (6 faces) gives $\chi^2 = 15$ with $df = 5$ and p-value 0.01.

- (a) State the degrees of freedom.
- (b) State the decision at $\alpha = 0.05$.
- (c) State the conclusion in context.

Solution 3.3

Method: The procedure

Worked steps

(a) 5 **B1**. (b) reject H_0 **B1**. (c) there is convincing evidence the die does not follow the claimed (fair) distribution **B1**.