Exercise walk-through

Carrying Out a Test for the Difference of Two Population Means ▪ 7.9

eXercise

You must be able to

- compute the two-sample t statistic and find the **p-value**
- compare the p-value to α and reach a decision
- state a conclusion in context

Method — The procedure

1. State $H_0 : \mu_1 = \mu_2$ and H_a; check conditions.
2. Compute $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$.
3. Find the **p-value** from the t-distribution.
4. Compare to α and decide (reject or fail to reject).
5. State the conclusion **in context**.

A **large** $|t|$ gives a small p-value — stronger evidence of a difference.

Method — A worked test

$\bar{x}_1 = 20, s_1 = 3, n_1 = 25$ and $\bar{x}_2 = 18, s_2 = 4, n_2 = 25$. Then

$$t = \frac{20 - 18}{\sqrt{3^2/25 + 4^2/25}} = \frac{2}{\sqrt{1}} = 2.0.$$

This gives a p-value near 0.05 – borderline evidence of a difference in means.

Practice 2.1 ▪ 2 marks

State the steps to carry out a two-sample t-test.

Solution 2.1

Worked steps

state hypotheses and check conditions; compute t; find the p-value; compare to α and conclude B1 B1 *for two steps; for the full procedure.*

Practice 2.2 ▪ 1 mark

State what a large $|t|$ suggests.

Solution 2.2

Worked steps

a difference between the two means (a small p-value) B1.

Practice 2.3 ▪ 1 mark

A two-sample t-test gives a p-value of 0.06 at $\alpha = 0.05$. State the decision.

Solution 2.3

Worked steps

fail to reject H_0 $(0.06 > 0.05)$ **B1**.

Exam-style 3.1 ▪ 1 mark

A two-sample t-test compares

A two proportions

B two means

C one mean to a fixed value

D categories

Solution 3.1

A two proportions

B two means ✓

C one mean to a fixed value

D categories

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

A two-sample t-test fails to reject H_0 when the p-value is

A below α

B above α

C exactly zero

D negative

Solution 3.2

A below α

B above α ✓

C exactly zero

D negative

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

A two-sample t-test gives a p-value of 0.01 at $\alpha = 0.05$.
(a) State the decision.
(b) State whether the means differ significantly.
(c) State the conclusion in context.

Solution 3.3

Method: The procedure

Worked steps

(a) reject H_0 B1. (b) yes B1. (c) there is convincing evidence that the two means differ B1.