

Exercise walk-through

Setting Up a Test for the Difference of Two Population Means ■ 7.8

eXercise

You must be able to

- state the hypotheses $H_0 : \mu_1 = \mu_2$ and H_a

- write the test statistic $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

Method — Hypotheses

- **Null** $H_0 : \mu_1 = \mu_2$ (no difference).
- **Alternative** $H_a : \mu_1 \neq \mu_2$ (or one-sided).

Method — Test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}.$$

The standard error again adds the two **variances** (s^2/n).

Practice 2.1 ■ 1 mark

State the null hypothesis for a two-sample mean test.

Solution 2.1

Worked steps

$$H_0 : \mu_1 = \mu_2 \quad \text{B1}.$$

Practice 2.2 ■ 1 mark

Write the test-statistic formula for a difference of means.

Solution 2.2

Method: Test statistic

Worked steps

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} \quad \text{B1}.$$

Practice 2.3 ■ 2 marks

For $\bar{x}_1 = 50$, $s_1 = 6$, $n_1 = 36$, $\bar{x}_2 = 45$, $s_2 = 8$, $n_2 = 64$, find the standard error

$$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}.$$

Solution 2.3

Method: Test statistic

Worked steps

$$\sqrt{\frac{36}{36} + \frac{64}{64}} = \sqrt{1 + 1} = \sqrt{2} \approx 1.41 \quad \text{M1 A1}.$$

Exam-style 3.1 ■ 1 mark

The null hypothesis for a two-sample mean test is

A $\mu_1 \neq \mu_2$

B $\mu_1 = \mu_2$

C $\mu_1 = 0$

D $\bar{x}_1 = \bar{x}_2$

Solution 3.1

A $\mu_1 \neq \mu_2$

B $\mu_1 = \mu_2$ 

C $\mu_1 = 0$

D $\bar{x}_1 = \bar{x}_2$

Worked steps

B.

Exam-style 3.2 ■ 1 mark

The test statistic for a difference of means uses the standard error

A $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

B $s_1 + s_2$

C $s_1 \cdot s_2$

D $\sqrt{np}$

Solution 3.2

Method: Test statistic

A $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$



B $s_1 + s_2$

C $s_1 \cdot s_2$

D $\sqrt{np}$

Worked steps

A.

Exam-style 3.3 ■ 2 marks

$\bar{x}_1 = 70$, $s_1 = 10$, $n_1 = 25$, $\bar{x}_2 = 65$, $s_2 = 10$, $n_2 = 25$.

- (a) Find the standard error.
- (b) Compute the test statistic t .

Solution 3.3

Method: Test statistic

Worked steps

$$(a) \sqrt{\frac{100}{25} + \frac{100}{25}} = \sqrt{8} \approx 2.83 \quad \text{M1 A1}. \quad (b) \quad t = \frac{70 - 65}{2.83} \approx 1.77 \quad \text{M1 A1}.$$