

Past-paper walk-through

Carrying Out a Test for a Population Mean ■ 7.5

eXercise

Questions in this set

- 2023 Q4
- 2018 Q6

Reading the mark scheme

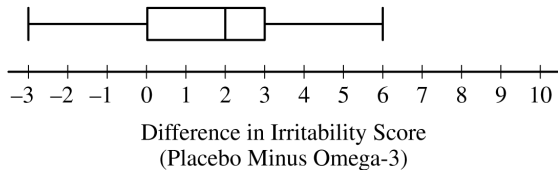
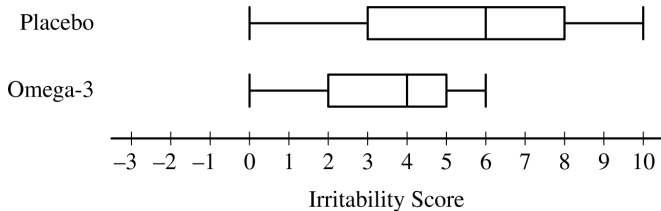
- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- **//** separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 4

2023 ■ Q4

(a) The researcher claims the omega-3 supplement will decrease the mean irritability score of all patients with the medical condition similar to the volunteers who participated in the study. Is there convincing statistical evidence to support the researcher's claim at a significance level of $\alpha = 0.05$? Complete the appropriate inference procedure to support your answer.

Question 4



Solution 4

(a) Mark scheme

- This inference question is scored holistically across THREE sections (each rated essentially correct/partially correct/incorrect, then combined via a lookup table into an integer 0-4), not as separately-added marks. Full model solution:
- SECTION 1 (hypotheses + test identification): Let μ_d represent the true mean difference (placebo minus omega-3) of irritability scores for all people with this medical condition. $H_0: \mu_d = 0$. $H_a: \mu_d > 0$. The appropriate inference procedure is a matched-pairs (paired) t-test for a mean difference. Full credit for this section needs: (1) identifies a paired t-test for a population mean difference by name or formula, (2) states the hypotheses using a single mean (e.g. μ_d), (3) states the correct null equality ($\mu_d=0$) AND the correct direction for the

Solution 4

alternative ($\mu_d > 0$), (4) provides sufficient context for the parameter - population mean difference AND the sampling units (people with the medical condition) AND the response variable (irritability score).

- SECTION 2 (conditions + test statistic + p-value): The independence condition is satisfied because the data came from a randomized experiment - the week each patient received each treatment was randomly assigned. The sampling distribution of the mean difference is approximately normal: although $n=19 < 30$, the boxplot of sample differences is approximately symmetric with no outliers, so this condition is satisfied. Test statistic: $t = (\bar{x}_d - \mu_0)/(s/\sqrt{n}) = (1.789 - 0)/(2.485/\sqrt{19}) \approx 3.138$. Using 18 degrees of freedom, the p-value is $P(t > 3.138) \approx 0.0028$. Full credit needs: (1) checks independence by referring to the random assignment, (2) indicates the sample differences are not badly skewed, supporting approximate normality of the sampling distribution of $\bar{x}_d$, (3) correctly reports the t-statistic

Solution 4

(≈ 3.138) consistent with the named test, (4) correctly reports the p-value (≈ 0.0028) consistent with the stated alternative and test statistic.

- SECTION 3 (conclusion): Because the p-value (≈ 0.0028) is less than $\alpha = 0.05$, the null hypothesis should be rejected. The data provide convincing statistical evidence that for patients similar to those in the study, the true mean difference (placebo minus omega-3) in irritability scores for people with this medical condition is greater than zero - suggesting omega-3 fatty acids are helpful in reducing irritability scores in people with this medical condition. Full credit needs: (1) correct comparison of the p-value to alpha AND a correct decision about the null/alternative hypothesis, (2) a conclusion stated in context, consistent with the alternative hypothesis, using non-deterministic language (e.g. 'convincing evidence', never 'proves').

Solution 4

- Grade holistically: 3 sections essentially correct $\rightarrow$ 4/4 (Complete); 2 EC + 1 P $\rightarrow$ 3/4 (Substantial); 2 EC + 0 P, OR 1 EC + 1-2 P, OR 3 P $\rightarrow$ 2/4 (Developing); 1 EC + 0 P, OR 0 EC + 2 P $\rightarrow$ 1/4 (Minimal). Equivalent confidence-interval or two-sample approaches (if independence between the two treatment groups is justified) may also earn full credit provided the corresponding interval/statistic and conclusion are computed and interpreted consistently.

Question 6

2018 ■ Q6

Blood pressure is the amount of pressure that blood exerts on blood vessels while the heart is beating. The mean systolic blood pressure for people in the United States is reported to be 122 millimeters of mercury (mmHg) with a standard deviation of 15 mmHg.

The wellness department of a large corporation is investigating whether the mean systolic blood pressure of its employees is greater than the reported national mean. A random sample of 100 employees will be selected, and the systolic blood pressure of each employee in the sample will be measured, and the sample mean will be calculated. Let μ represent the mean systolic blood pressure of all employees at the corporation. Consider the following hypotheses.

$$H_0 : \mu = 122 \quad H_a : \mu > 122$$

Question 6

(a) Describe a Type II error in the context of the hypothesis test.

Question 6

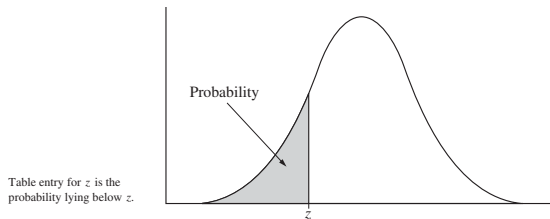


Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183

Question 6

-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Question 6

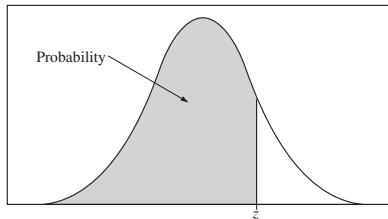


Table entry for z is the probability lying below z .

Table A (Continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015

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[illegible]

Question 6

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^* .

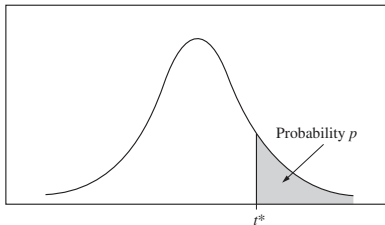


Table B t distribution critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587

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10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%

Confidence level C

Question 6

Table entry for p is the point (χ^2) with probability p lying above it.

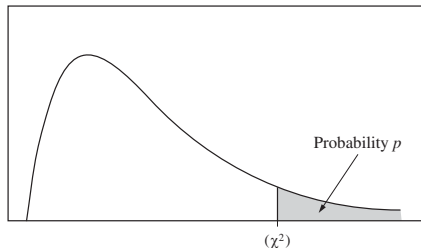


Table C χ^2 critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87

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2018 ■ Q6

Blood pressure is the amount of pressure that blood exerts on blood vessels while the heart is beating. The mean systolic blood pressure for people in the United States is reported to be 122 millimeters of mercury (mmHg) with a standard deviation of 15 mmHg.

The wellness department of a large corporation is investigating whether the mean systolic blood pressure of its employees is greater than the reported national mean. A random sample of 100 employees will be selected, and the systolic blood pressure of each employee in the sample will be measured, and the sample mean will be calculated.

Let μ represent the mean systolic blood pressure of all employees at the corporation. Consider the following hypotheses.

$$H_0 : \mu = 122 \quad H_a : \mu > 122$$

Question 6

(b) Assume that σ , the standard deviation of the systolic blood pressure of all employees at the corporation, is 15 mmHg. If $\mu = 122$, the sampling distribution of $\bar{x}$ for samples of size 100 is approximately normal with a mean of 122 mmHg and a standard deviation of 1.5 mmHg. What values of the sample mean $\bar{x}$ represent sufficient evidence to reject the null hypothesis at the significance level of $\alpha = 0.05$?

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2018 ■ Q6

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Let μ represent the mean systolic blood pressure of all employees at the corporation. Consider the following hypotheses.

$$H_0 : \mu = 122 \quad H_a : \mu > 122$$

Question 6

(c) The actual mean systolic blood pressure of all employees at the corporation is 125 mmHg, and the standard deviation is 15 mmHg. Using the actual mean of 125 mmHg and the results from part (b), determine the probability that the null hypothesis would be rejected.

Question 6

2018 ■ Q6

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Let μ represent the mean systolic blood pressure of all employees at the corporation. Consider the following hypotheses.

$$H_0 : \mu = 122 \quad H_a : \mu > 122$$

(d) What statistical term is used for the probability found in part (c) ?

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2018 ■ Q6

Blood pressure is the amount of pressure that blood exerts on blood vessels while the heart is beating. The mean systolic blood pressure for people in the United States is reported to be 122 millimeters of mercury (mmHg) with a standard deviation of 15 mmHg.

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Let μ represent the mean systolic blood pressure of all employees at the corporation. Consider the following hypotheses.

$$H_0 : \mu = 122 \quad H_a : \mu > 122$$

Question 6

(e) Suppose the size of the sample of employees selected at the corporation is greater than 100. Would the probability of rejecting the null hypothesis be greater than, less than, or equal to the probability calculated in part (c) ? Explain your reasoning.

Solution 6

(a) Mark scheme

- A Type II error occurs when the alternative hypothesis is actually true, but the null hypothesis is not rejected. In context: a Type II error would happen if the true mean systolic blood pressure of the POPULATION of employees at the corporation is actually greater than 122 mmHg (the national average), but the hypothesis test fails to reject $H_0 : \mu = 122$ — i.e., the test does not detect/conclude that the employees' mean blood pressure is higher than the national average even though it truly is. Full credit requires the description to include, in context (units/blood pressure/employees referenced): (1) that the alternative hypothesis is true, and (2) that the null hypothesis is not rejected. A

Solution 6

description that refers to an individual employee's blood pressure rather than the mean of the population of employees does not satisfy either component.

Solution 6

(b) Mark scheme

- This is a one-sided (upper-tail) z-test with known σ . Reject H_0 when the test statistic $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} > 1.645$ (the critical value for the upper 5

Solution 6

(c) Mark scheme

- If the actual population mean is $\mu = 125$ mmHg (with $\sigma = 15$, $n = 100$), the sampling distribution of $\bar{x}$ is approximately Normal with mean 125 and standard deviation $15/\sqrt{100} = 1.5$. Using the rejection threshold from part (b), $\bar{x} > 124.4675$:

$$P(\bar{x} > 124.4675) = P\left(z > \frac{124.4675 - 125}{1.5}\right) = P(z > -0.355) \approx 0.64. \text{ Full}$$

credit requires: (1) recognizing that H_0 is rejected when $\bar{x} \geq 124.4675$ (an incorrect threshold carried forward from part (b) may still be used), (2) the correct sampling distribution of $\bar{x}$ when the true mean is 125 — correct mean and

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SD, given explicitly or via the test-statistic formula, and (3) using the normal curve to find the correct resulting probability (≈ 0.64).

Solution 6

(d)

Mark scheme

- The probability found in part (c) — the probability of correctly rejecting the (false) null hypothesis when the true mean is 125 — is called the POWER of the test. This is a brief vocabulary identification; the AP rubric scores it jointly with part (e) as a single section, so it carries no separate mark value of its own (its credit is folded into part (e)) — but a correct response must still name the term 'power' specifically (not 'confidence level', 'significance level', or similar).

Solution 6

(e) Mark scheme

- If the sample size is increased beyond 100 (true mean still 125), the probability of (correctly) rejecting H_0 — the power — would be GREATER than the value found in part (c) (≈ 0.64). Reasoning: the rejection region stays $z > 1.645$, but a larger n makes the standard deviation of the sampling distribution of $\bar{x}$ smaller (since $\sigma_{\bar{x}} = \sigma/\sqrt{n}$); a smaller $\sigma_{\bar{x}}$ means the minimum value of $\bar{x}$ needed to reject H_0 (the value at $z = 1.645$) is lower, closer to $\mu_0 = 122$ — so more of the sampling distribution centered at 125 falls in the rejection region, and the probability of rejecting H_0 increases. Full credit (shared across parts d and e, 4 components total) requires: naming 'power' as the term from part (c) [part d]; and, for part (e), correctly stating the probability would be greater, correctly

Solution 6

implying that the standard deviation of the sampling distribution decreases (explicitly or by substituting into the formula), and correctly indicating that the minimum $\bar{x}$ value needed to reject H_0 decreases.