

Exercise walk-through

Carrying Out a Test for a Population Mean ■ 7.5

eXercise

You must be able to

- compute the t test statistic and find the **p-value** from the t-distribution
- compare the p-value to α and reach a decision
- state a conclusion in context

Method — The procedure

- 1 State $H_0 : \mu = \mu_0$ and H_a ; check conditions.
- 2 Compute $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$ with $df = n - 1$.
- 3 Find the **p-value** from the **t-distribution**.
- 4 Compare to α : reject H_0 if $p\text{-value} < \alpha$, else fail to reject.
- 5 State the conclusion **in context**.

Method — A worked test

Test $H_0 : \mu = 50$ against $H_a : \mu \neq 50$ with $\bar{x} = 52$, $s = 4$, $n = 16$. Then

$$t = \frac{52 - 50}{4/\sqrt{16}} = \frac{2}{1} = 2.0, \quad \text{df} = 15.$$

The two-sided p-value $\approx 0.06 > 0.05$, so we fail to reject H_0 .

Practice 2.1 ■ 2 marks

State the steps to carry out a t-test for a mean.

Solution 2.1

Worked steps

state hypotheses and check conditions; compute t ; find the p-value; compare to α and conclude **B1** **B1** *for two steps; for the full procedure.*

Practice 2.2 ■ 1 mark

State the distribution that gives the p-value.

Solution 2.2

Worked steps

the t-distribution **B1**.

Practice 2.3 ■ 1 mark

A t-test gives a p-value of 0.04 with $\alpha = 0.05$. State the decision.

Solution 2.3

Worked steps

reject H_0 ($0.04 < 0.05$) **B1**.

Exam-style 3.1 ■ 1 mark

The p-value for a mean test comes from the

- A** normal distribution
- B** t-distribution
- C** binomial distribution
- D** χ^2 distribution

Solution 3.1

A normal distribution

B t-distribution 

C binomial distribution

D χ^2 distribution

Worked steps

B.

Exam-style 3.2 ■ 1 mark

A t-test rejects H_0 when the p-value is

- A** above α
- B** below α
- C** exactly zero
- D** exactly one

Solution 3.2

- A above α
- B below α
- C exactly zero
- D exactly one



Worked steps

B.

Exam-style 3.3 ■ 1 mark

A t-test of $\mu > 100$ gives a p-value of 0.02 at $\alpha = 0.05$.

- (a) State the decision.
- (b) State whether the result is significant.
- (c) State the conclusion in context.

Solution 3.3

Method: The procedure

Worked steps

(a) reject H_0 **B1**. (b) yes **B1**. (c) there is convincing evidence that $\mu > 100$ **B1**.