

Exercise walk-through

Introducing Statistics: Should I Worry About Error? ■ 7.1

eXercise

You must be able to

- explain why inference for a mean uses the **t-distribution** t 分布 when σ is unknown
- state that the sample standard deviation s estimates σ
- describe how the t -distribution approaches the normal as n grows

Method — Why t, not z

When the population standard deviation σ is **unknown** (almost always), we estimate it with the **sample** standard deviation s . This extra uncertainty means we use the **t-distribution** instead of the normal.

Method — The t-distribution

It is symmetric but has **heavier tails** than the normal, with **degrees of freedom** $= n - 1$. As n grows, it approaches the normal distribution.

Practice 2.1 ■ 1 mark

State why we use the t-distribution instead of z for means.

Solution 2.1

Method: Why t, not z

Worked steps

because σ is unknown and estimated by s , adding uncertainty **B1**.

Practice 2.2 ■ 1 mark

State what replaces σ when it is unknown.

Solution 2.2

Method: Why t , not z

Worked steps

the sample standard deviation s **B1**.

Practice 2.3 ■ 1 mark

State what happens to the t-distribution as n increases.

Solution 2.3

Worked steps

it approaches the normal distribution **B1**.

Exam-style 3.1 ■ 1 mark

When the population σ is unknown, inference for a mean uses the

- A** z-distribution
- B** t-distribution
- C** binomial distribution
- D** uniform distribution

Solution 3.1

Method: Why t, not z

A z-distribution

B t-distribution 

C binomial distribution

D uniform distribution

Worked steps

B.

Exam-style 3.2 ■ 1 mark

The t-distribution is

- A** narrower than the normal
- B** wider than the normal (heavier tails)
- C** always identical to the normal
- D** skewed

Solution 3.2

- A narrower than the normal
- B wider than the normal (heavier tails)** 
- C always identical to the normal
- D skewed

Worked steps

B.

Exam-style 3.3 ■ 1 mark

A sample of size 20 estimates a mean, with σ unknown.

- (a) Name the distribution used.
- (b) State what estimates σ .
- (c) State the degrees of freedom.

Solution 3.3

Method: Why t, not z

Worked steps

(a) the t-distribution **B1**. (b) the sample standard deviation s **B1**. (c) $n - 1 = 19$ **B1**.