Past-paper walk-through

Setting Up a Test for a Population Proportion ▪ 6.4

eXercise

Questions in this set

- 2025 Q4
- 2021 Q4

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

2025 ▪ Q4

A software application (app) lets users enter questions to receive answers in the form of images, texts, or videos. Research indicates that 22 percent of high school students in Country W use the app to help them with their homework at least once per week. Karen is an AP Statistics student in Country W at a high school that has more than 2,000 students. She believes the proportion of all students at her school who use the app to help them with their homework at least once per week is greater than the proportion for her country. To investigate her belief, she took a simple random sample of 130 students from her school and found that 38 of the sampled students use the app to help them with their homework at least once per week.

Is there convincing statistical evidence, at a 0.05 significance level, to support Karen's belief? Justify your answer with the appropriate inference procedure.

Solution 4

- This question is scored holistically across three sections (each E/P/I, combined via a lookup table into an integer 0–4), not as separately-added marks — the full model solution: SECTION 1 (test identification + hypotheses): the appropriate procedure is a one-sample z-test for a population proportion. Let p = the true proportion of all students at Karen's high school who use the app to help with homework at least once per week. H0: $p = 0.22$, Ha: $p > 0.22$ (one-sided, since Karen believes the true proportion at her school is greater than the 22% reported for Country W). SECTION 2 (conditions + test statistic + p-value): Independence — the sample of 130 students is random, and $130 \leq 0.10(2000)$ since the school has more than 2,000 students (sampling without replacement

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from a large population). Large Counts — the expected numbers of successes and failures are both at least 10: n · p0 = 130(0.22) = 28.6 and n(1−p0) = 130(0.78) = 101.4, so the sampling distribution of $\hat{p}$ is approximately normal. With 38 of the 130 sampled students using the app at least weekly, $\hat{p} = 38/130 \approx 0.2923$. Test statistic: z = (0.2923 − 0.22) / sqrt(0.22(1−0.22)/130) ≈ 1.99. p-value = P(Z > 1.99) ≈ 0.023. SECTION 3 (conclusion): Because the p-value (≈0.023) is less than the significance level $\alpha = 0.05$, the null hypothesis should be rejected — there is convincing statistical evidence that the true proportion of students at Karen's high school who use the app to help with homework at least once per week is greater than Country W's proportion of 0.22. (An equivalent confidence-interval approach: a one-sided 95% confidence interval for p is (0.227, ∞), or a two-sided 90% CI is (0.227, 0.358); since 0.22 falls below the lower

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bound, this supports the same rejection of H0 — note a two-sided 95% CI of (0.214, 0.370) does NOT by itself support rejecting at α=0.05 one-sided.) Grade holistically: full 4/4 needs correct hypotheses+test ID, correct conditions-check+test-statistic+p-value, and a correct conclusion in context using nondefinitive language (e.g. 'convincing evidence', not 'proves'); partial credit for partially-correct sections per the AP holistic table (3 sections correct→4, 2 correct+1 partial→3, etc.).

Question 4

2021 ▪ Q4

The manager of a large company that sells pet supplies online wants to increase sales by encouraging repeat purchases. The manager believes that if past customers are offered $10 off their next purchase, more than 40 percent of them will place an order. To investigate the belief, 90 c mail with a coupon for 10 off the next purchase if the order is placed within 30 days. Of those who receive the coupon, 38 place an order.

(a) Is there convincing statistical evidence, at the significance level of $\alpha = 0.05$, that the manager's belief is correct? Complete the appropriate inference procedure to support your answer.

(b) Based on your conclusion from part (a), which of the two errors, Type I or Type II, could have been made? Interpret the consequence of the error in context.

Solution 4

(a) Mark scheme

- Let $p =$ the true proportion of all customers of the pet supply company who would place an order within 30 days after receiving the coupon e-mail.
  $H_0 : p = 0.40$, $H_a : p > 0.40$. Appropriate procedure: one-proportion z-test.
  Conditions: the 90 customers are a random sample, and the company is assumed to have at least $10(90) = 900$ customers (10

Solution 4

(b) Mark scheme

- Because H_0 was not rejected in part (a), a Type II error could have been made – failing to reject a false null hypothesis. Consequence in context: if in fact more than 40 percent of all customers would actually place an order after receiving the coupon e-mail (i.e., H_0 is really false), but the test failed to detect this, the manager may decide NOT to use the coupon promotion even though it would actually result in more than 40 percent of customers placing an order within 30 days – a missed profitable opportunity. Credit requires (1) correctly stating a Type II error could have been made (or Type I, if part (a) rejected H_0), AND (2) a reasonable, sufficiently contextualized interpretation of the consequence (referencing customers placing an order / the coupon / sales).