Exercise walk-through

Constructing a Confidence Interval for a Population Proportion ▪ 6.2

eXercise

You must be able to

- construct a confidence interval $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
- use the appropriate **critical value** z^*
- interpret the interval

Method — The confidence interval

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}},$$

where the **margin of error** is z^* times the standard error. For 95% confidence, $z^* = 1.96$.

Method — Interpretation

"We are 95% confident that the true proportion p lies within the interval."

Method — Example

$$\hat{p} = 0.6,\ n = 100,\ 95\%\text{: ME} = 1.96\sqrt{\frac{0.6(0.4)}{100}} = 1.96(0.049) = 0.096.$$

Practice 2.1 ▪ 1 mark

Write the formula for a confidence interval for a proportion.

Solution 2.1

Worked steps

$\hat{p} \pm z^* \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$ B1.

Practice 2.2 ▪ 1 mark

State the z^* value for a 95% confidence interval.

Solution 2.2

Method: The confidence interval

Worked steps

1.96 B1.

Practice 2.3 ▪ 2 marks

A poll gives $\hat{p} = 0.5$ with $n = 100$. Find the 95% margin of error.

Solution 2.3

Method: The confidence interval

Worked steps

$$\text{ME} = 1.96\sqrt{\frac{0.5(0.5)}{100}} = 1.96(0.05) = 0.098$$ M1 A1.

Exam-style 3.1 ▪ 1 mark

A confidence interval for p has the form

A $\hat{p} \pm \text{ME}$

B $\hat{p} \times \text{ME}$

C $\hat{p} / \text{ME}$

D ME only

Solution 3.1

Method: The confidence interval

A $\hat{p} \pm \text{ME}$ ✓

B $\hat{p} \times \text{ME}$

C $\hat{p}/\text{ME}$

D ME only

Worked steps

A.

Exam-style 3.2 ▪ 1 mark

For a 95% confidence interval, z^* is about

A 1.28

B 1.645

C 1.96

D 2.58

Solution 3.2

Method: The confidence interval

A 1.28

B 1.645

C 1.96 ✓

D 2.58

Worked steps

C.

Exam-style 3.3 ▪ 2 marks

$\hat{p} = 0.4$, $n = 100$, 95% confidence.
(a) Find the margin of error.
(b) Give the confidence interval.

Solution 3.3

Method: The confidence interval

Worked steps

(a) $\text{ME} = 1.96\sqrt{\dfrac{0.4(0.6)}{100}} = 1.96(0.049) = 0.096$ M1 A1. (b) $(0.304,\ 0.496)$ B1.