

Exercise walk-through

Carrying Out a Test for the Difference of Two Population Proportions ■ 6.11

eXercise

You must be able to

- compute the test statistic z using the pooled standard error
- find the **p-value** and compare it to α
- state a conclusion in context

Method — The full procedure

1 State $H_0 : p_1 = p_2$ and H_a ; check conditions.

2 Compute $z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_c(1 - \hat{p}_c) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$.

3 Find the **p-value** from z .

4 Compare to α : reject H_0 if p-value $< \alpha$, else fail to reject.

5 State the conclusion **in context**.

Method — A worked test

$\hat{p}_1 = 0.60$ ($n_1 = 50$), $\hat{p}_2 = 0.45$ ($n_2 = 60$). Pooled $\hat{p}_c = \frac{30+27}{110} = 0.518$. Then

$$z = \frac{0.60 - 0.45}{\sqrt{0.518(0.482) \left(\frac{1}{50} + \frac{1}{60} \right)}} \approx \frac{0.15}{0.096} \approx 1.57,$$

a two-sided p-value $\approx 0.12 > 0.05$ – fail to reject H_0 .

Practice 2.1 ■ 2 marks

State the steps to carry out a two-proportion test.

Solution 2.1

Worked steps

state hypotheses and check conditions; compute z ; find the p-value; compare to α and conclude in context **B1** **B1** *for two steps; for the full procedure.*

Practice 2.2 ■ 1 mark

A two-proportion test gives $z = 2.5$. State whether this is likely significant at $\alpha = 0.05$.

Solution 2.2

Worked steps

yes — $z = 2.5$ gives a small p-value, likely below 0.05 **B1**.

Practice 2.3 ■ 1 mark

State what to compare the p-value to.

Solution 2.3

Worked steps

the significance level α **B1**.

Exam-style 3.1 ■ 1 mark

After computing z , you find the

A slope

B p-value

C mean

D residual

Solution 3.1

A slope

B p-value



C mean

D residual

Worked steps

B.

Exam-style 3.2 ■ 1 mark

A two-proportion test rejects H_0 when the p-value is

- A** above α
- B** below α
- C** exactly zero
- D** exactly one

Solution 3.2

- A above α
- B below α
- C exactly zero
- D exactly one



Worked steps

B.

Exam-style 3.3 ■ 1 mark

A two-proportion test gives a p-value of 0.03 at $\alpha = 0.05$.

- (a) State the decision.
- (b) State whether the two proportions differ significantly.
- (c) State the conclusion in context.

Solution 3.3

Method: The full procedure

Worked steps

(a) reject H_0 **B1**. (b) yes **B1**. (c) there is convincing evidence that the two proportions differ **B1**.