Exercise walk-through

Setting Up a Test for the Difference of Two Population Proportions ▪ 6.10

eXercise

You must be able to

- state the hypotheses $H_0 : p_1 = p_2$ and H_a
- explain why a **pooled (combined) proportion 合并比例** is used
- write the pooled proportion $\hat{p}_c = \dfrac{x_1 + x_2}{n_1 + n_2}$

Method — Hypotheses

- **Null** $H_0 : p_1 = p_2$ (no difference).
- **Alternative** $H_a : p_1 \neq p_2$ (or one-sided).

Method — The pooled proportion

Because H_0 assumes the proportions are **equal**, we combine both samples to estimate that common value:

$$\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2},$$

and use $\hat{p}_c$ in the standard error of the test statistic.

Practice 2.1 ▪ 1 mark

State the null hypothesis for a two-proportion test.

Solution 2.1

Method: The pooled proportion

Worked steps

$H_0 : p_1 = p_2$ B1.

Practice 2.2 ▪ 1 mark

State why a pooled proportion is used in the test.

Solution 2.2

Method: The pooled proportion

Worked steps

H_0 assumes the proportions are equal, so both samples estimate the same common value B1.

Practice 2.3 ▪ 1 mark

Write the formula for the pooled proportion.

Solution 2.3

Method: The pooled proportion

Worked steps

$$\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$$ B1.

Exam-style 3.1 ▪ 1 mark

The null hypothesis in a two-proportion test is

A $p_1 \neq p_2$

B $p_1 = p_2$

C $p_1 = 0$

D $\hat{p}_1 = \hat{p}_2$

Solution 3.1

Method: The pooled proportion

A $p_1 \neq p_2$

B $p_1 = p_2$ ✓

C $p_1 = 0$

D $\hat{p}_1 = \hat{p}_2$

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

The pooled proportion combines

A only group 1

B both groups' successes and sample sizes

C the two means

D nothing

Solution 3.2

Method: The pooled proportion

A only group 1

B both groups' successes and sample sizes ✓

C the two means

D nothing

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

Group 1 has 30 successes in 100; group 2 has 20 in 100.
(a) Find $\hat{p}_1$.
(b) Find $\hat{p}_2$.
(c) Find the pooled proportion $\hat{p}_c$.

Solution 3.3

Method: The pooled proportion

Worked steps

(a) 0.30 B1. (b) 0.20 B1. (c) $\hat{p}_c = \dfrac{30 + 20}{100 + 100} = 0.25$ B1.