Past-paper walk-through

Sampling Distributions for Sample Means ▪ 5.7

eXercise

Questions in this set

- 2026 Q6
- 2023 Q6
- 2015 Q6
- 2014 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 6

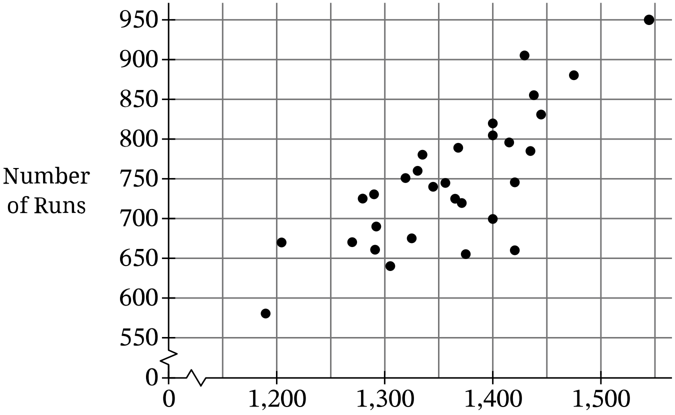
2026 ▪ Q6

The following information applies to parts A and B.

6. To win a game, a baseball team needs to score runs. To score runs, players need to get on base. The primary way players get on base is by hitting the ball. Figure 1 shows the relationship between the number of hits and the number of runs for 30 randomly selected professional baseball teams.

Figure 1. Scatterplot of Number of Runs by Number of Hits

Question 6


Number of Runs
950
900
850
800
750
700
650
600
550
0
0
1,200
1,300
1,400
1,500

Question 6

Number of Hits

Part A

Consider the scatterplot in Figure 1.

i. Describe the relationship between the number of hits and the number of runs for these baseball teams, in context.

ii. Based on the data from the sample of 30 teams, an equation of the least-squares regression line model for predicting the number of runs from the number of hits is as follows.

$$\text{Predicted number of runs} = -372.2 + 0.823\,(\text{number of hits})$$

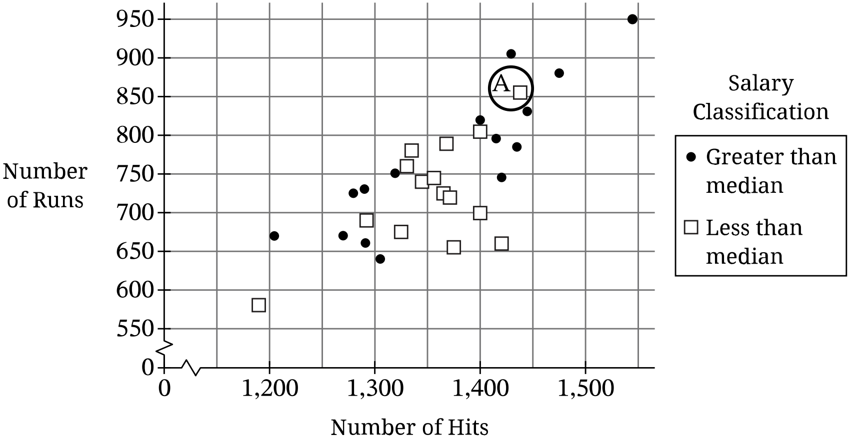
Using the given regression equation, calculate the point estimate for the predicted number of runs a team would score if they were to achieve their goal of 1,250 hits. Show your work.

Part B

Each team has a total salary equal to the sum of the salaries of all players on the team. The median total salary for the sample of 30 teams is $160 million. In Figure 2, points with dots represent teams with a total salary greater than the median, and points with squares represent teams with a total salary less than the median.

Figure 2. Scatterplot of Number of Runs by Number of Hits and Salary Classification

Question 6


Number of Runs
950
900
850
800
750
700
650
600
550
0
0
1,200
1,300
1,400
1,500
Number of Hits
A
Salary Classification
Greater than median
Less than median

Question 6

Consider the scatterplot in Figure 2.

i. Compare the team represented by the point that is circled and labeled A with the other teams that have the same total salary classification.

ii. For each salary classification, consider the linear relationship between the number of hits and the number of runs. For teams with a total salary greater than the median, is the strength of the linear relationship stronger than, weaker than, or similar to the strength of the linear relationship for teams with a total salary less than the median? Explain.

Part C

Different types of intervals can be used when working with linear regression models. One type of interval is a confidence interval for the slope of the least-squares regression line. Two other types of intervals in the context of this problem are as follows:

Question 6

- A confidence interval that estimates the mean number of runs for all teams with a specific number of hits.
- A prediction interval that predicts the number of runs for a single team with a specific number of hits.

All three intervals use the same basic formula:

$$\text{Point Estimate} \pm t^{*}(\text{standard error}),$$

where critical value t^{*} comes from a t-distribution that has $n-2$ degrees of freedom. The same critical value is used when finding each of these intervals with the same level of confidence.

Consider the point estimate for the predicted number of runs found in part A (ii) for the team whose goal is to achieve 1,250 hits next year. Recall that this value was calculated using the linear regression model based on the data from the sample of 30 teams.

Question 6

i. What is the critical value for a 95% confidence level that would be used for the confidence interval for the mean number of runs as well as for the prediction interval for the number of runs? Indicate the answer to two decimal places.

ii. The standard error for the confidence interval for the mean number of runs is 17.48. Assuming the conditions for inference are met, calculate the 95% confidence interval for the mean number of runs for all teams with 1,250 hits. Show your work.

iii. The standard error for the prediction interval for the number of runs is 56.78. Assuming the conditions for inference are met, calculate the 95% prediction interval for the number of runs for a single team with 1,250 hits. Show your work.

Part D

i. Would a distribution of sample means be expected to have more variability or less variability than a distribution of individual observations? Explain.

Question 6

ii. The standard error used in the confidence interval that estimates the mean number of runs for all teams with x hits can be found using the following formula.

$$\sqrt{s^2\left(\frac{1}{n}+\frac{(x-\overline{x})^2}{\sum(x_i-\overline{x})^2}\right)}$$

The standard error used in the prediction interval that predicts the number of runs for a single team with x hits can be found using the following formula.

$$\sqrt{s^2+s^2\left(\frac{1}{n}+\frac{(x-\overline{x})^2}{\sum(x_i-\overline{x})^2}\right)}$$

In both standard error formulas, s is the standard deviation of the residuals, n is the sample size, and $\overline{x}$ is the sample mean number of hits. Based on the answer from part D (i) and the standard error formulas for the confidence interval and the prediction interval, explain why the prediction interval calculated in part C (iii) is wider than the confidence interval calculated in part C (ii).

STOP

END OF EXAM

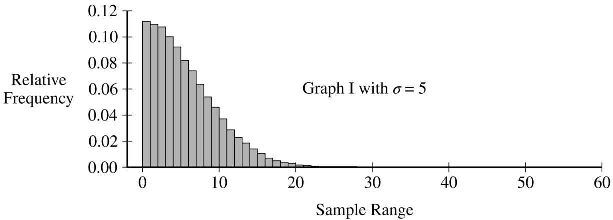
Question 6

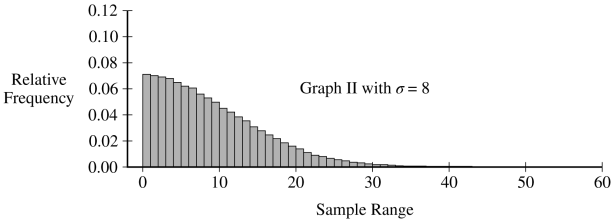
2023 ▪ Q6

A jewelry company uses a machine to apply a coating of gold on a certain style of necklace. The amount of gold applied to a necklace is approximately normally distributed. When the machine is working properly, the amount of gold applied to a necklace has a mean of 300 milligrams (mg) and standard deviation of 5 mg.

(a) A necklace is randomly selected from the necklaces produced by the machine. Assuming that the machine is working properly, calculate the probability that the amount of gold applied to the necklace is between 296 mg and 304 mg.

Question 6

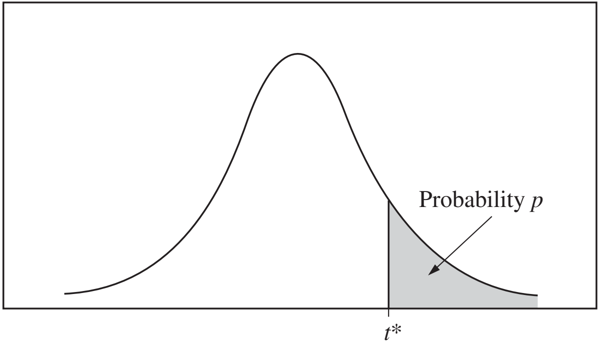

0.12
0.10
0.08
0.06
0.04
0.02
0.00
Relative Frequency
Graph I with $\sigma = 5$
0
10
20
30
40
50
60
Sample Range


0.12
0.10
0.08
0.06
0.04
0.02
0.00
Relative Frequency
Graph II with $\sigma = 8$
0
10
20
30
40
50
60
Sample Range

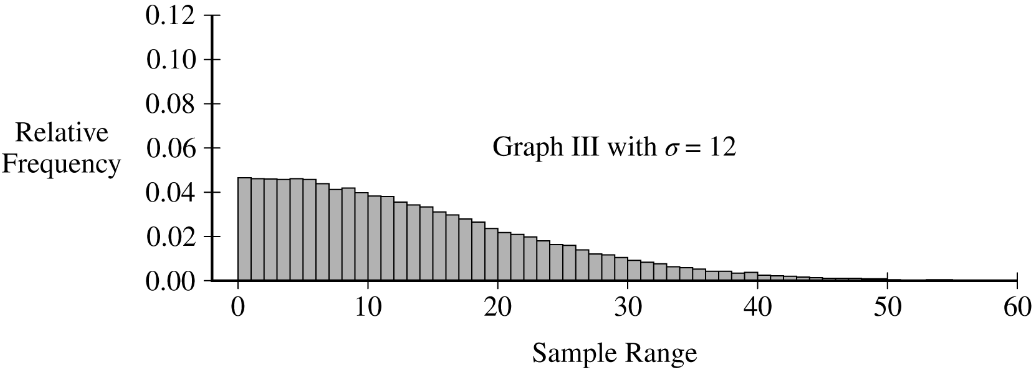
Question 6

3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.



Question 6



Relative Frequency
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Graph III with $\sigma = 12$
0
10
20
30
40
50
60
Sample Range

Question 6

2023 ▪ Q6

A jewelry company uses a machine to apply a coating of gold on a certain style of necklace. The amount of gold applied to a necklace is approximately normally distributed. When the machine is working properly, the amount of gold applied to a necklace has a mean of 300 milligrams (mg) and standard deviation of 5 mg.

(b)(i) The jewelry company wants to make sure the machine is working properly. Each day, Cleo, a statistician at the jewelry company, will take a random sample of the necklaces produced that day. Each selected necklace will be melted down and the amount of the gold applied to that necklace will be determined. Because a necklace must be destroyed to determine the amount of gold that was applied, Cleo will use random samples of size $n = 2$ necklaces.

Question 6

Cleo starts by considering the mean amount of gold being applied to the necklaces. After Cleo takes a random sample of $n = 2$ necklaces, she computes the sample mean amount of gold applied to the two necklaces.
Suppose the machine is working properly with a population mean amount of gold being applied of 300 mg and a population standard deviation of 5 mg.
Calculate the probability that the sample mean amount of gold applied to a random sample of $n = 2$ necklaces will be greater than 303 mg.
(b)(ii) Suppose Cleo took a random sample of $n = 2$ necklaces that resulted in a sample mean amount of gold applied of 303 mg. Would that result indicate that the population mean amount of gold being applied by the machine is different from 300 mg? Justify your answer without performing an inference procedure.

Question 6

2023 ▪ Q6

A jewelry company uses a machine to apply a coating of gold on a certain style of necklace. The amount of gold applied to a necklace is approximately normally distributed. When the machine is working properly, the amount of gold applied to a necklace has a mean of 300 milligrams (mg) and standard deviation of 5 mg.

(c)(i) Now, Cleo will consider the variation in the amount of gold the machine applies to the necklaces. Because of the small sample size, $n = 2$, Cleo will use the sample range of the data for the two randomly selected necklaces, rather than the sample standard deviation.

Cleo will investigate the behavior of the range for samples of size $n = 2$. She will simulate the sampling distribution of the range of the amount of gold applied to two randomly sampled necklaces. Cleo generates $100{,}000$ random samples of size $n = 2$ independent values from a normal distribution with mean $\mu = 300$ and standard

Question 6

deviation $\sigma = 5$. The range is calculated for the two observations in each sample. The simulated sampling distribution of the range is shown in Graph I. This process is repeated using $\sigma = 8$, as shown in Graph II, and again using $\sigma = 12$, as shown in Graph III.

[Graph I with $\sigma = 5$: histogram titled "Graph I with $\sigma = 5$"; x-axis "Sample Range" from 0 to 60; y-axis "Relative Frequency" from 0.00 to 0.12. Distribution is strongly right-skewed, peak (about 0.11-0.12) near sample range 0-2, decreasing steadily to near 0 by about range 25-30.]

[Graph II with $\sigma = 8$: histogram titled "Graph II with $\sigma = 8$"; x-axis "Sample Range" from 0 to 60; y-axis "Relative Frequency" from 0.00 to 0.12. Distribution is right-skewed, peak (about 0.07) near sample range 0-2, decreasing steadily to near 0 by about range 40-45.]

Question 6

[Graph III with $\sigma = 12$: histogram titled "Graph III with $\sigma = 12$"; x-axis "Sample Range" from 0 to 60; y-axis "Relative Frequency" from 0.00 to 0.12. Distribution is right-skewed, peak (about 0.047) near sample range 0-2, decreasing gradually, with a longer tail extending out to about range 55-60.]

Use the information in the graphs to complete the following.

Describe the sampling distribution of the sample range for random samples of size $n = 2$ from a normal distribution with standard deviation $\sigma = 5$, as shown in Graph I.

(c)(ii) Describe how the sampling distribution of the sample range for samples of size $n = 2$ changes as the value of the population standard deviation σ increases.

Question 6

2023 ▪ Q6

A jewelry company uses a machine to apply a coating of gold on a certain style of necklace. The amount of gold applied to a necklace is approximately normally distributed. When the machine is working properly, the amount of gold applied to a necklace has a mean of 300 milligrams (mg) and standard deviation of 5 mg.

(d)(i) Recall that Cleo needs to consider both the mean and standard deviation of the amount of gold applied to necklaces to determine whether the machine is working properly. Suppose that one month later, Cleo is again checking the machine to make sure it is working properly. Cleo takes a random sample of 2 necklaces and calculates the sample mean amount of gold applied as 303 mg and the sample range as 10 mg. Recall that the machine is working properly if the amount of gold applied to the necklaces has a mean of 300 mg and standard deviation of 5 mg.

Question 6

Consider Cleo's range of 10 mg from the sample of size $n = 2$. If the machine is working properly with a standard deviation of 5 mg, is a sample range of 10 mg unusual? Justify your answer.

(d)(ii) Do Cleo's sample mean of 303 mg and range of 10 mg indicate that the machine is not working properly? Explain your answer.

Solution 6

(a) Mark scheme

- Let X represent the amount of gold (mg) applied to a randomly selected necklace. Assuming the machine works properly, X has an approximately normal distribution with mean 300 mg and standard deviation 5 mg. $P(296 < X < 304) = P(X{<}304) - P(X{\leq}296) = P(Z < (304{-}300)/5) - P(Z \leq (296{-}300)/5) = P(Z{<}0.8) - P(Z{\leq}{-}0.8) \approx 0.7881 - 0.2119 \approx 0.5763$. Full credit needs all 3 components: (1) indicates use of a normal distribution with mean 300 and standard deviation 5 (may be shown graphically, via calculator normalcdf syntax, in words, in standard notation N(300,5), or via a labeled z-score calculation), (2) specifies the correct event (boundary values 296 and 304, correct direction) or an event consistent with component 1, (3) provides the correct probability 0.5763, or a probability

Solution 6

consistent with components 1 and 2. Partial credit if only 2 of the 3 components are satisfied. Minor errors in statistical notation may be ignored (though considered poor communication under holistic scoring). If the only error is a reversed z-score numerator (300—296 or 300—304), the response is scored partially correct.

Solution 6

(b)(i) Mark scheme

- If the machine is working properly, the sample mean amount of gold $\bar{X}$ (for a sample of n=2 necklaces) has a sampling distribution that is approximately normal with mean 300 mg and standard deviation $5/\sqrt{2} \approx 3.5355$ mg. Then $P(\bar{X} > 303) = P(Z > (303-300)/3.5355) \approx P(Z > 0.8485) \approx 0.198$. NOTE: parts (b)(i) and (b)(ii) are scored TOGETHER as ONE holistic unit in the official scoring guidelines, so this leaf carries this part's full mark. Full credit for the combined part (b) needs components 1, 4, and 5 AND at least one of components 2 or 3: (1) in part (b)(i), indicates use of a normal/approximately-normal distribution and identifies the correct parameter values (mean 300, SD $5/\sqrt{2}\approx3.5355$) with work shown or a correct formula, (2) in part (b)(i), specifies the correct event (boundary

Solution 6

value and direction: $\bar{X}>303$), (3) in part (b)(i), provides the correct probability 0.198 (or one consistent with components 1-2), (4) in part (b)(ii), indicates whether observing a sample mean of 303 mg would provide convincing evidence the population mean is not 300 mg, consistent with the response to (b)(i) [graded together with leaf 6(b)(ii)], (5) in part (b)(ii), provides justification based on the probability obtained in (b)(i) or a correctly-computed probability in (b)(ii). An alternate solution using the TOTAL amount of gold on the 2 necklaces (N(600, 7.071), P(total>606)) earning full component-1/2/3 credit is also acceptable.

Solution 6

(b)(ii) Mark scheme

- Observing a sample mean amount of 303 mg would NOT provide convincing evidence that the population mean amount of gold being applied by the machine is something other than 300 mg, because the probability of observing a sample mean that differs from 300 mg by 3 mg or more (in either direction) is large - around $0.198(2) = 0.396$ - so a difference this size is not statistically surprising if the machine were working properly. This leaf's credit is bundled with 6(b)(i) in the official scoring guidelines (Part (b) of the SG scores both sub-parts together as one holistic unit), so it carries 0 of this question's marks on its own - but a correct, well-justified 'no' answer, consistent with the probability found in (b)(i), is still expected to complete credit for part (b).

Solution 6

(c)(i) Mark scheme

- Cleo simulates the sampling distribution of the sample range (max−min) for random samples of size n=2 from a normal distribution with standard deviation σ=5 mg (100,000 simulated samples). The resulting sampling distribution of the sample range is skewed to the right. Almost all simulated values of the range fall between 0 mg and about 25-30 mg, and the center of the distribution is about 6 mg. NOTE: parts (c)(i) and (c)(ii) are scored TOGETHER as ONE holistic unit in the official scoring guidelines, so this leaf carries this part's full mark. Full credit for the combined part (c) needs 4 or 5 of the following 5 components: (1) in part (c)(i), describes the shape as positively skewed (skewed right), (2) in part (c)(i), describes the center of the distribution as about 6 mg (any value between 4

Solution 6

and 10 mg is a reasonable center description), (3) in part (c)(i), describes the spread as having most values between 0 mg and 30 mg (or IQR between 5 and 7 mg inclusive), (4) in part (c)(ii), indicates the mean/center of the sample-range distribution increases as the population SD σ increases [graded together with leaf 6(c)(ii)], (5) in part (c)(ii), indicates the variation/spread of the sample-range distribution also increases as σ increases. Partial credit if only 3 of the 5 components are satisfied.

Solution 6

(c)(ii) Mark scheme

- As the value of the population standard deviation σ increases, the variation (spread) in the distribution of the sample range increases, and the mean (center) of the distribution of the sample range also increases. This leaf's credit is bundled with 6(c)(i) in the official scoring guidelines (Part (c) of the SG scores both sub-parts together as one holistic unit), so it carries 0 of this question's marks on its own - but stating BOTH that the center increases AND that the spread increases (as σ increases) is still expected to complete credit for part (c). Comments on skewness are not required here and should be treated as extraneous.

Solution 6

(d)(i) Mark scheme

- No, a sample range of 10 mg is not unusual if the machine is working properly with a standard deviation of 5 mg. Although observing a sample range around 10 mg or greater is much more likely if the population SD is 8 mg or 12 mg, the sampling distribution of the sample range (n=2) for a normal distribution with σ=5 mg shows that 10 mg is not an unusual value - there is about a 20% chance that a random sample of 2 necklaces would yield a range of 10 mg or more when the machine is working properly. NOTE: parts (d)(i) and (d)(ii) are scored TOGETHER as ONE holistic unit in the official scoring guidelines, so this leaf carries this part's full mark. Full credit for the combined part (d) needs 4 or 5 of the following 5 components: (1) in part (d)(i), indicates a sample range of 10 mg

Solution 6

is not unusual, (2) in part (d)(i), justifies this by indicating the σ=5 sampling-distribution graph shows values of 10 mg or greater occur fairly often, (3) in part (d)(ii), indicates there is not convincing evidence the machine is not working properly [graded together with leaf 6(d)(ii)], (4) in part (d)(ii), justifies this by indicating a sample mean of 303 mg would not be unusual (e.g. it is less than one SD from 300, or consistent with the $\approx$0.198 probability from part (b)), (5) in part (d)(ii), justifies this by indicating a sample range of 10 mg is not unusual, consistent with (d)(i). Partial credit if at least 3 of the 5 components are satisfied.

Solution 6

(d)(ii) Mark scheme

- No, Cleo's sample mean of 303 mg and range of 10 mg do NOT indicate that the machine is not working properly. As noted in part (b)(i), the probability that the sample mean would be equal to or greater than 303 mg when the machine is working properly is almost 20%, so a sample mean of 303 mg is not unusual (it is also less than one standard deviation, $5/\sqrt{2} \approx 3.5355$ mg, away from 300 mg). As indicated in part (d)(i), the probability of a range of 10 mg or greater when the population SD is 5 mg is also about 20%, so it too is not unusual. There is not statistically significant evidence to show the machine is not working properly. This leaf's credit is bundled with 6(d)(i) in the official scoring guidelines (Part (d) of the SG scores both sub-parts together as one holistic unit), so it carries 0 of this

Solution 6

question's marks on its own - but this correct 'no' conclusion, justified by BOTH the mean's and the range's probabilities not being unusual (consistent with (d)(i)), is still expected to complete credit for part (d). Context (amount of gold in necklaces) is not required in parts (a)-(d) but is considered good communication.

Question 6

2015 ▪ Q6

Corn tortillas are made at a large facility that produces 100,000 tortillas per day on each of its two production lines. The distribution of the diameters of the tortillas produced on production line A is approximately normal with mean 5.9 inches, and the distribution of the diameters of the tortillas produced on production line B is approximately normal with mean 6.1 inches. The figure below shows the distributions of diameters for the two production lines.
[Two overlapping bell-shaped normal distribution curves labeled “Production Line A” (solid curve, centered at 5.9) and “Production Line B” (dashed curve, centered at 6.1), sharing a horizontal axis “Diameter (inches)” marked 5.7, 5.8, 5.9, 6.0, 6.1, 6.2, 6.3, and a vertical axis labeled “Probability”.]
The tortillas produced at the two production lines are advertised as having a diameter of 6 inches. For the purpose of quality control, a sample of 200 tortillas is selected per

Question 6

day at each corporation, and the diameters are measured. From the sample of 200 tortillas, the manager of the facility wants to estimate the mean diameter, in inches, of the 200,000 tortillas produced on a given day. Two sampling methods have been proposed.

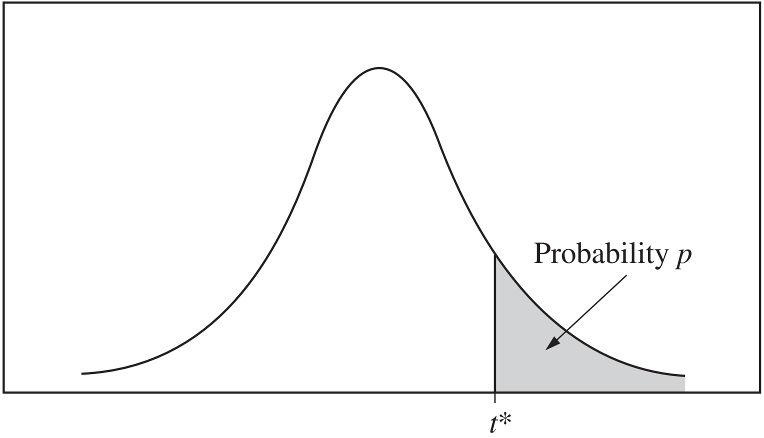
Method 1: Take a random sample of 200 tortillas from the 200,000 tortillas produced on a given day. Measure the diameter of each selected tortilla.

Method 2: Randomly select one of the two production lines on a given day. Take a random sample of 200 tortillas from the 100,000 tortillas produced by the selected production line. Measure the diameter of each selected tortilla.

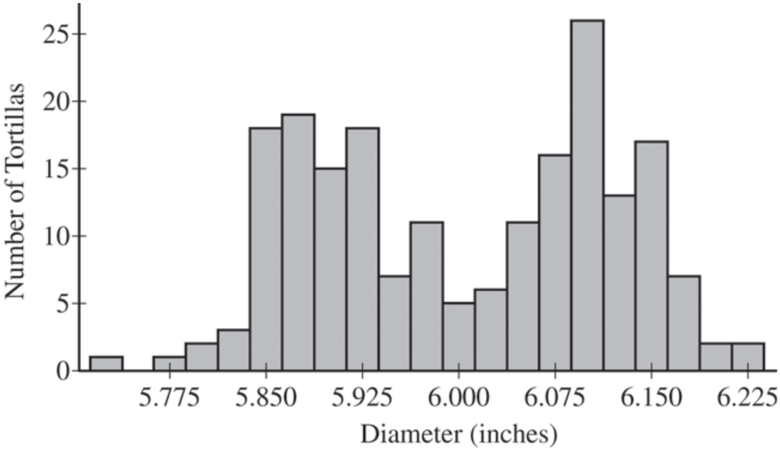
(a) Will a sample obtained by using Method 2 be representative of the population of all tortillas made that day, with respect to the diameters of the tortillas? Why or why not.

Question 6

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.



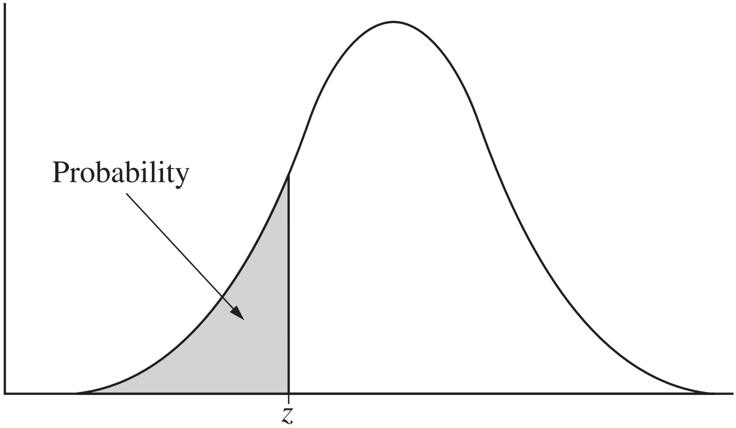
Question 6



Number of Tortillas
25
20
15
10
5
0
5.775
5.850
5.925
6.000
6.075
6.150
6.225
Diameter (inches)

Question 6

Table entry for z is the probability lying below z.



Question 6

2015 ▪ Q6

Corn tortillas are made at a large facility that produces 100,000 tortillas per day on each of its two production lines. The distribution of the diameters of the tortillas produced on production line A is approximately normal with mean 5.9 inches, and the distribution of the diameters of the tortillas produced on production line B is approximately normal with mean 6.1 inches. The figure below shows the distributions of diameters for the two production lines.
[Two overlapping bell-shaped normal distribution curves labeled "Production Line A" (solid curve, centered at 5.9) and "Production Line B" (dashed curve, centered at 6.1), sharing a horizontal axis "Diameter (inches)" marked 5.7, 5.8, 5.9, 6.0, 6.1, 6.2, 6.3, and a vertical axis labeled "Probability".]
The tortillas produced at the two production lines are advertised as having a diameter of 6 inches. For the purpose of quality control, a sample of 200 tortillas is selected per day at each corporation, and the diameters are measured. From the sample of 200 tortillas, the manager of

Question 6

the facility wants to estimate the mean diameter, in inches, of the 200,000 tortillas produced on a given day. Two sampling methods have been proposed.

Method 1: Take a random sample of 200 tortillas from the 200,000 tortillas produced on a given day. Measure the diameter of each selected tortilla.

Method 2: Randomly select one of the two production lines on a given day. Take a random sample of 200 tortillas from the 100,000 tortillas produced by the selected production line. Measure the diameter of each selected tortilla.

(b) The figure below is a histogram of 200 diameters obtained by using one of the two sampling methods described. Choosing the shape of the histogram as a guide, explain which method, Method 1 or Method 2, was most likely used to obtain a such a sample. [Histogram titled with y-axis "Number of Tortillas" from 0 to 25 and x-axis "Diameter (inches)" with bins at approximately 5.775, 5.850, 5.925, 6.000, 6.075, 6.150, 6.225. The histogram is bimodal, with one cluster of taller bars centered around 5.850-5.925

Question 6

(peak near 20-22) and a second cluster of bars centered around 6.075-6.150 (peak near 20-22), with a smaller gap/dip around 6.000.]

Question 6

2015 ▪ Q6

Corn tortillas are made at a large facility that produces 100,000 tortillas per day on each of its two production lines. The distribution of the diameters of the tortillas produced on production line A is approximately normal with mean 5.9 inches, and the distribution of the diameters of the tortillas produced on production line B is approximately normal with mean 6.1 inches. The figure below shows the distributions of diameters for the two production lines.
[Two overlapping bell-shaped normal distribution curves labeled "Production Line A" (solid curve, centered at 5.9) and "Production Line B" (dashed curve, centered at 6.1), sharing a horizontal axis "Diameter (inches)" marked 5.7, 5.8, 5.9, 6.0, 6.1, 6.2, 6.3, and a vertical axis labeled "Probability".]
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Question 6

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Method 1: Take a random sample of 200 tortillas from the 200,000 tortillas produced on a given day. Measure the diameter of each selected tortilla.

Method 2: Randomly select one of the two production lines on a given day. Take a random sample of 200 tortillas from the 100,000 tortillas produced by the selected production line. Measure the diameter of each selected tortilla.

(c) Which of the two sampling methods, Method 1 or Method 2, will result in less variability in the diameters of the 200 tortillas on the sample on a given day? Explain why.

Question 6

2015 ▪ Q6

Corn tortillas are made at a large facility that produces 100,000 tortillas per day on each of its two production lines. The distribution of the diameters of the tortillas produced on production line A is approximately normal with mean 5.9 inches, and the distribution of the diameters of the tortillas produced on production line B is approximately normal with mean 6.1 inches. The figure below shows the distributions of diameters for the two production lines.
[Two overlapping bell-shaped normal distribution curves labeled "Production Line A" (solid curve, centered at 5.9) and "Production Line B" (dashed curve, centered at 6.1), sharing a horizontal axis "Diameter (inches)" marked 5.7, 5.8, 5.9, 6.0, 6.1, 6.2, 6.3, and a vertical axis labeled "Probability".]
The tortillas produced at the two production lines are advertised as having a diameter of 6 inches. For the purpose of quality control, a sample of 200 tortillas is selected per day at each corporation, and the diameters are measured. From the sample of 200 tortillas, the manager of

Question 6

the facility wants to estimate the mean diameter, in inches, of the 200,000 tortillas produced on a given day. Two sampling methods have been proposed.

Method 1: Take a random sample of 200 tortillas from the 200,000 tortillas produced on a given day. Measure the diameter of each selected tortilla.

Method 2: Randomly select one of the two production lines on a given day. Take a random sample of 200 tortillas from the 100,000 tortillas produced by the selected production line. Measure the diameter of each selected tortilla.

(d)(i) Each day, the distribution of the 200,000 tortillas made that day has mean diameter 6 inches with standard deviation 0.11 inch.

For samples of size 200 taken from one day's production, describe the sampling distribution of the sample mean diameter for samples that are obtained using Method 1.

Question 6

(d)(ii) Suppose that one of the two sampling methods will be selected and used every day for one year (365 days). The sample mean of the 200 diameters will be recorded each day. Which of the two methods will result in less variability in the distribution of the 365 sample means? Explain.

Question 6

2015 ▪ Q6

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(e) A government inspector will visit the factory on June 22 to observe the sampling and to determine if the factory is in compliance with the advertised mean diameter of 6 inches. The manager knows that, with both sampling methods, the sample mean is an unbiased estimator of the population mean. However, the manager is unsure which method is more likely to produce a sample mean that is close to 6 inches on the day of sampling. Based on your previous answers, which of the two sampling methods,

Question 6

Method 1 or Method 2, is more likely to produce a sample mean close to 6 inches? Explain.

Solution 6

(a)

Mark scheme

- No. A sample obtained using Method 2 will not be representative of the population of all tortillas made that day - it represents only the tortillas from one production line, not the whole population, because the diameter distributions differ between the two production lines.

Solution 6

(b)

Mark scheme

- Method 1 was most likely used. The bimodal shape of the histogram shows tortillas were selected from both production lines, consistent with Method 1; Method 2 would tend to produce a unimodal distribution centered at either 5.9 or 6.1 inches, not the observed bimodal shape.

Solution 6

(c)

Mark scheme

- Method 2 would give less variability in the sample of 200 tortillas on a given day, because that sample comes from only one production line. Since the two lines' diameter distributions differ, sampling from both lines (Method 1) produces more variable sample data.

Solution 6

(d)(i)

Mark scheme

- The sampling distribution of the sample mean diameter for samples of size 200 obtained using Method 1 is approximately Normal, with mean 6 inches and standard deviation $\dfrac{0.11}{\sqrt{200}} \approx 0.0078$ inch.

Solution 6

(d)(ii)

Mark scheme

- Method 1 would give less variability in the 365 sample means. With Method 2, roughly half of the 365 daily sample means will be close to 5.9 inches (production line A) and the other half close to 6.1 inches (production line B) - a bimodal spread. With Method 1, the sample means will all be close to 6 inches, tightly clustered as shown by the small standard deviation (≈ 0.0078 inch) from part (d).

Solution 6

(e)

Mark scheme

- Method 1. On a single day, Method 1 is more likely to produce a sample mean close to the true mean of 6 inches. Since the sample mean is an unbiased estimator of the population mean under both methods, the manager should pick the method with less variability in the distribution of the sample mean - which, based on the previous part, is Method 1.

Question 3

2014 ▪ Q3

Schools in a certain state receive funding based on the number of students who attend the school. To determine the number of students who attend a school, one school day is selected at random and the number of students in attendance that day is counted and used for funding purposes. The daily number of absences at High School A in the state is approximately normally distributed with mean of 120 students and standard deviation of 10.5 students.

(a) If more than 140 students are absent on the day the attendance count is taken for funding purposes, the school will lose some of its state funding in the subsequent year. Approximately what is the probability that High School A will lose some state funding?

Question 3

Each trial in the simulation consists of rolling three fair, six-sided dice, one die for each of the convention attendees. For each die, rolling a 1, 2, 3, or 4 represents selecting a man; rolling a 5 or 6 represents selecting a woman. After 1,000 trials, the number of times the dice indicate selecting 3 women is recorded.

Question 3

2014 ▪ Q3

Schools in a certain state receive funding based on the number of students who attend the school. To determine the number of students who attend a school, one school day is selected at random and the number of students in attendance that day is counted and used for funding purposes. The daily number of absences at High School A in the state is approximately normally distributed with mean of 120 students and standard deviation of 10.5 students.

(b) The principals' association in the state suggests that instead of choosing one day at random, the state should choose 3 days at random. With the suggested plan, High School A would lose some of its state funding in the subsequent year if the mean number of students absent for the 3 days is greater than 140. Would High School A be more likely, less likely, or equally likely to lose funding using the suggested plan compared to the plan described in part (a)? Justify your choice.

Question 3

2014 ▪ Q3

Schools in a certain state receive funding based on the number of students who attend the school. To determine the number of students who attend a school, one school day is selected at random and the number of students in attendance that day is counted and used for funding purposes. The daily number of absences at High School A in the state is approximately normally distributed with mean of 120 students and standard deviation of 10.5 students.

(c) A typical school week consists of the days Monday, Tuesday, Wednesday, Thursday, and Friday. The principal at High School A believes that the number of absences tends to be greater on Mondays and Fridays, and there is concern that the school will lose state funding if the attendance count occurs on a Monday or Friday. If one school day is chosen at random from each of 3 typical school weeks, what is the probability that none of the 3 days chosen is a Tuesday, Wednesday, or Thursday?

Solution 3

(a) Mark scheme

- (Q3 is graded holistically across parts (a)-(c) via a combination table into an overall 0-4 score; this leaf's marks approximate its share of the 4-point total.) The daily number of absences is approximately normal with mean 120 and standard deviation 10.5, so the z-score for 140 absences is $z = (140\text{-}120)/10.5 \approx 1.90$. The standard normal table/calculator gives $P(X \leq 140) = 0.9713$, so P(more than 140 absent, i.e. the school loses funding) $= 1 - 0.9713 = 0.0287$. Full credit needs three components: (1) indicates use of a normal distribution and clearly identifies the correct parameter values (showing correct components of a z-score calculation is sufficient), (2) uses a correct boundary value (140, 140.5, or 141 all acceptable), (3) reports the correct normal probability consistent with (1) and (2)

Solution 3

— OR reports a probability of 0.025 justified via the empirical rule with an acceptable boundary value. Partial credit for correctly providing only two of the three components, or for reporting an incorrect probability of 0.05 justified via the empirical rule with an acceptable boundary value. An inconsistency between calculation steps lowers the score for this part by one level.

Solution 3

(b) Mark scheme

- High School A would be LESS likely to lose state funding under the suggested 3-day plan. With a random sample of 3 days, the distribution of the sample mean number of absences has less variability than the distribution of a single day's absences, concentrating more narrowly around the mean of 120 and giving a smaller probability that it exceeds 140. Specifically, the standard deviation of the sample mean is $\sigma/\sqrt{n} = 10.5/\sqrt{3} \approx 6.062$, so the z-score for a sample mean of 140 is $(140\text{-}120)/6.062 \approx 3.30$, giving a probability of $1 - 0.9995 = 0.0005$ of losing funding under the suggested plan — less than the 0.0287 probability found in part (a). (An equivalent approach uses the total absences over 3 days: mean $3(120){=}360$, sd $3(6.062){\approx}18.187$, $z{=}(420\text{-}360)/18.187{\approx}3.30$, giving the same

Solution 3

probability; a response using this total-based approach is scored E if it gives the correct answer and references the distribution of the sample total with the correct mean and standard deviation.) Full credit needs the correct answer 'less likely' AND three components: (1) clearly references the distribution of the sample mean, (2) indicates the variability of that distribution is smaller (than for a single day), (3) indicates the distribution is centered at 120 — OR the correct answer 'less likely' AND both: correctly calculates the probability the sample mean exceeds 140 (arithmetic errors not penalized) AND correctly compares this probability to part (a)'s. Partial credit for 'less likely' with only two of the first three components, or for 'less likely' with a correctly-calculated exceedance probability but no explicit comparison to part (a). Incorrect if the response does not give the correct answer of 'less likely' with adequate support, including a correct answer given with no explanation or an incorrect explanation.

Solution 3

(c) Mark scheme

- For any one typical school week (Monday-Friday), P(the day selected is not Tuesday, Wednesday, or Thursday) $= 2/5 = 0.4$ (only Monday or Friday qualify). Because the days are selected independently across the three weeks, P(none of the 3 selected days is a Tuesday, Wednesday, or Thursday) $= (0.4)\hat{}3 = 0.064$. Full credit needs the response to correctly calculate the probability AND show sufficient supporting work. Partial credit for reporting the correct probability (0.064) with no work shown or insufficient work shown, or for using the multiplication rule involving the three weekly selections but doing so incorrectly and/or with an incorrect single-week probability of not selecting a Tuesday, Wednesday, or Thursday.