Exercise walk-through

Sampling Distributions for Differences in Sample Proportions ▪ 5.6

eXercise

You must be able to

- use the mean $\mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2$
- use the standard deviation $\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$

Method — Difference in proportions

To compare two groups, use $\hat{p}_1 - \hat{p}_2$:

$$\mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2, \qquad \sigma = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}.$$

Notice the two variances **add** under the square root (the samples are independent).

Method — A worked value

Two machines have defect rates $p_1 = 0.10$ ($n_1 = 100$) and $p_2 = 0.06$ ($n_2 = 150$). Then

$$\mu = 0.10 - 0.06 = 0.04, \qquad \sigma = \sqrt{\frac{0.10(0.90)}{100} + \frac{0.06(0.94)}{150}} \approx \sqrt{0.001276} \approx 0.036.$$

Practice 2.1 ▪ 1 mark

Write the mean of the sampling distribution of $\hat{p}_1 - \hat{p}_2$.

Solution 2.1

Worked steps

$p_1 - p_2$ B1.

Practice 2.2 ▪ 2 marks

For $p_1 = 0.5$, $n_1 = 50$, $p_2 = 0.4$, $n_2 = 50$, find the standard deviation.

Solution 2.2

Worked steps

$$\sqrt{\frac{0.5(0.5)}{50}+\frac{0.4(0.6)}{50}}=\sqrt{0.005+0.0048}=\sqrt{0.0098}=0.099 \quad \text{M1}\ \text{A1}.$$

Practice 2.3 ▪ 1 mark

State what this sampling distribution is used to compare.

Solution 2.3

Worked steps

the difference between two population proportions B1.

Exam-style 3.1 ▪ 1 mark

The mean of $\hat{p}_1 - \hat{p}_2$ is

A $p_1 \cdot p_2$

B $p_1 - p_2$

C $p_1 + p_2$

D 0

Solution 3.1

A $p_1 \cdot p_2$

B $p_1 - p_2$ ✓

C $p_1 + p_2$

D 0

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

The standard deviation of $\hat{p}_1 - \hat{p}_2$ combines the two variances by

A subtracting them

B adding them

C multiplying them

D dividing them

Solution 3.2

A subtracting them

B adding them ✓

C multiplying them

D dividing them

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

$p_1 = 0.6$, $n_1 = 100$, $p_2 = 0.5$, $n_2 = 100$.
(a) Find the mean of $\hat{p}_1 - \hat{p}_2$.
(b) Write the formula for its standard deviation.

Solution 3.3

Worked steps

(a) $0.6 - 0.5 = 0.1$ B1. (b) $\sqrt{\dfrac{p_1(1-p_1)}{n_1} + \dfrac{p_2(1-p_2)}{n_2}}$ B1.