Exercise walk-through

The Central Limit Theorem ▪ 5.3

eXercise

You must be able to

- state the **Central Limit Theorem 中心极限定理**: for large n, the sampling distribution of $\bar{x}$ is approximately normal
- use the guideline $n \geq 30$
- explain that this holds **regardless** of the population's shape

Method — The Central Limit Theorem

For a **large enough** sample (a common guideline is $n \geq 30$), the sampling distribution of the sample mean $\bar{x}$ is approximately **normal** — **whatever** the shape of the original population.

Method — Why it matters

It lets us use normal-based methods to make inferences about a population mean, even from skewed populations.

Practice 2.1 ▪ 1 mark

State what the Central Limit Theorem says about the distribution of $\bar{x}$.

Solution 2.1

Method: The Central Limit Theorem

Worked steps

for a large sample it is approximately normal B1.

Practice 2.2 ▪ 1 mark

State the common sample-size guideline for the CLT.

Solution 2.2

Method: The Central Limit Theorem

Worked steps

$n \geq 30$ B1.

Practice 2.3 ▪ 1 mark

State whether the CLT depends on the population's shape.

Solution 2.3

Method: The Central Limit Theorem

Worked steps

no — it holds regardless of the population's shape B1.

Exam-style 3.1 ▪ 1 mark

By the Central Limit Theorem, the sampling distribution of $\bar{x}$ is approximately

A skewed

B normal for large n

C uniform

D binomial

Solution 3.1

Method: The Central Limit Theorem

A skewed

B normal for large n ✓

C uniform

D binomial

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

A common sample-size guideline for the CLT is

A $n \geq 5$

B $n \geq 30$

C $n \geq 100$

D $n = 1$

Solution 3.2

Method: The Central Limit Theorem

A $n \geq 5$

B $n \geq 30$ ✓

C $n \geq 100$

D $n = 1$

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

A population is strongly skewed. Samples of size 50 are taken.
(a) State the approximate shape of the sampling distribution of $\bar{x}$.
(b) Name the theorem responsible.
(c) State whether the population itself must be normal.

Solution 3.3

Method: The Central Limit Theorem

Worked steps

(a) approximately normal B1. (b) the Central Limit Theorem B1. (c) no B1.