Past-paper walk-through

Introducing Statistics: Why Is My Sample Not Like Yours? ▪ 5.1

eXercise

Questions in this set

- 2023 Q6
- 2019 Q6

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

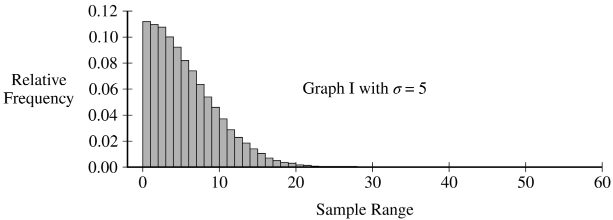
Question 6

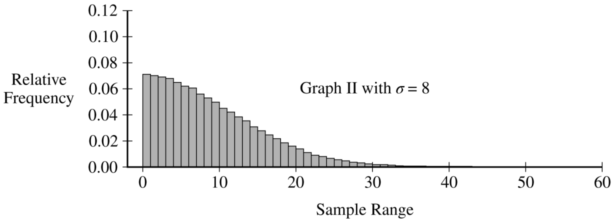
2023 ▪ Q6

A jewelry company uses a machine to apply a coating of gold on a certain style of necklace. The amount of gold applied to a necklace is approximately normally distributed. When the machine is working properly, the amount of gold applied to a necklace has a mean of 300 milligrams (mg) and standard deviation of 5 mg.

(a) A necklace is randomly selected from the necklaces produced by the machine. Assuming that the machine is working properly, calculate the probability that the amount of gold applied to the necklace is between 296 mg and 304 mg.

Question 6

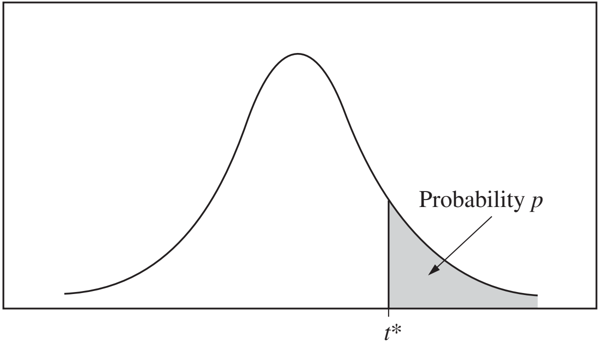

0.12
0.10
0.08
0.06
0.04
0.02
0.00
Relative Frequency
Graph I with $\sigma = 5$
0
10
20
30
40
50
60
Sample Range


0.12
0.10
0.08
0.06
0.04
0.02
0.00
Relative Frequency
Graph II with $\sigma = 8$
0
10
20
30
40
50
60
Sample Range

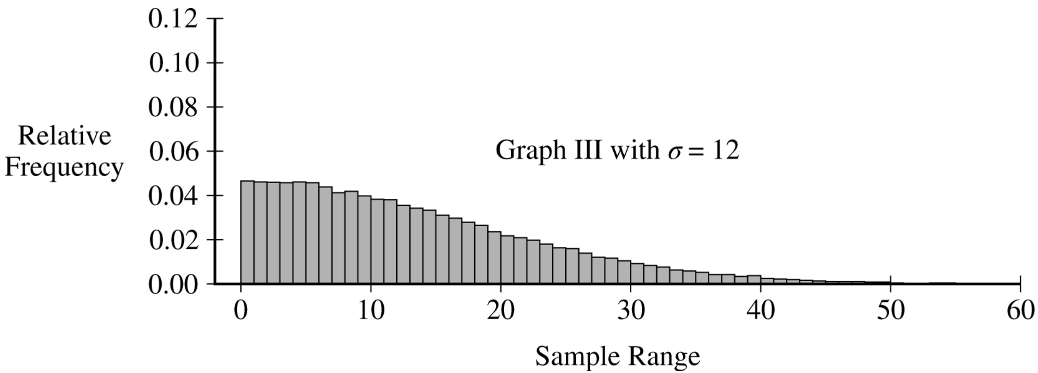
Question 6

3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.



Question 6



Relative Frequency
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Graph III with $\sigma = 12$
0
10
20
30
40
50
60
Sample Range

Question 6

2023 ▪ Q6

A jewelry company uses a machine to apply a coating of gold on a certain style of necklace. The amount of gold applied to a necklace is approximately normally distributed. When the machine is working properly, the amount of gold applied to a necklace has a mean of 300 milligrams (mg) and standard deviation of 5 mg.

(b)(i) The jewelry company wants to make sure the machine is working properly. Each day, Cleo, a statistician at the jewelry company, will take a random sample of the necklaces produced that day. Each selected necklace will be melted down and the amount of the gold applied to that necklace will be determined. Because a necklace must be destroyed to determine the amount of gold that was applied, Cleo will use random samples of size $n = 2$ necklaces.

Question 6

Cleo starts by considering the mean amount of gold being applied to the necklaces. After Cleo takes a random sample of $n = 2$ necklaces, she computes the sample mean amount of gold applied to the two necklaces.

Suppose the machine is working properly with a population mean amount of gold being applied of 300 mg and a population standard deviation of 5 mg.

Calculate the probability that the sample mean amount of gold applied to a random sample of $n = 2$ necklaces will be greater than 303 mg.

(b)(ii) Suppose Cleo took a random sample of $n = 2$ necklaces that resulted in a sample mean amount of gold applied of 303 mg. Would that result indicate that the population mean amount of gold being applied by the machine is different from 300 mg? Justify your answer without performing an inference procedure.

Question 6

2023 ▪ Q6

A jewelry company uses a machine to apply a coating of gold on a certain style of necklace. The amount of gold applied to a necklace is approximately normally distributed. When the machine is working properly, the amount of gold applied to a necklace has a mean of 300 milligrams (mg) and standard deviation of 5 mg.

(c)(i) Now, Cleo will consider the variation in the amount of gold the machine applies to the necklaces. Because of the small sample size, $n = 2$, Cleo will use the sample range of the data for the two randomly selected necklaces, rather than the sample standard deviation.

Cleo will investigate the behavior of the range for samples of size $n = 2$. She will simulate the sampling distribution of the range of the amount of gold applied to two randomly sampled necklaces. Cleo generates $100{,}000$ random samples of size $n = 2$ independent values from a normal distribution with mean $\mu = 300$ and standard

Question 6

deviation $\sigma = 5$. The range is calculated for the two observations in each sample. The simulated sampling distribution of the range is shown in Graph I. This process is repeated using $\sigma = 8$, as shown in Graph II, and again using $\sigma = 12$, as shown in Graph III.

[Graph I with $\sigma = 5$: histogram titled "Graph I with $\sigma = 5$"; x-axis "Sample Range" from 0 to 60; y-axis "Relative Frequency" from 0.00 to 0.12. Distribution is strongly right-skewed, peak (about 0.11-0.12) near sample range 0-2, decreasing steadily to near 0 by about range 25-30.]

[Graph II with $\sigma = 8$: histogram titled "Graph II with $\sigma = 8$"; x-axis "Sample Range" from 0 to 60; y-axis "Relative Frequency" from 0.00 to 0.12. Distribution is right-skewed, peak (about 0.07) near sample range 0-2, decreasing steadily to near 0 by about range 40-45.]

Question 6

[Graph III with $\sigma = 12$: histogram titled "Graph III with $\sigma = 12$"; x-axis "Sample Range" from 0 to 60; y-axis "Relative Frequency" from 0.00 to 0.12. Distribution is right-skewed, peak (about 0.047) near sample range 0-2, decreasing gradually, with a longer tail extending out to about range 55-60.]

Use the information in the graphs to complete the following.

Describe the sampling distribution of the sample range for random samples of size $n = 2$ from a normal distribution with standard deviation $\sigma = 5$, as shown in Graph I.

(c)(ii) Describe how the sampling distribution of the sample range for samples of size $n = 2$ changes as the value of the population standard deviation σ increases.

Question 6

2023 ▪ Q6

A jewelry company uses a machine to apply a coating of gold on a certain style of necklace. The amount of gold applied to a necklace is approximately normally distributed. When the machine is working properly, the amount of gold applied to a necklace has a mean of 300 milligrams (mg) and standard deviation of 5 mg.

(d)(i) Recall that Cleo needs to consider both the mean and standard deviation of the amount of gold applied to necklaces to determine whether the machine is working properly. Suppose that one month later, Cleo is again checking the machine to make sure it is working properly. Cleo takes a random sample of 2 necklaces and calculates the sample mean amount of gold applied as 303 mg and the sample range as 10 mg. Recall that the machine is working properly if the amount of gold applied to the necklaces has a mean of 300 mg and standard deviation of 5 mg.

Question 6

Consider Cleo's range of 10 mg from the sample of size $n = 2$. If the machine is working properly with a standard deviation of 5 mg, is a sample range of 10 mg unusual? Justify your answer.

(d)(ii) Do Cleo's sample mean of 303 mg and range of 10 mg indicate that the machine is not working properly? Explain your answer.

Solution 6

(a) Mark scheme

- Let X represent the amount of gold (mg) applied to a randomly selected necklace. Assuming the machine works properly, X has an approximately normal distribution with mean 300 mg and standard deviation 5 mg. $P(296 < X < 304) = P(X{<}304) - P(X{\leq}296) = P(Z < (304{-}300)/5) - P(Z \leq (296{-}300)/5) = P(Z{<}0.8) - P(Z{\leq}{-}0.8) \approx 0.7881 - 0.2119 \approx 0.5763$. Full credit needs all 3 components: (1) indicates use of a normal distribution with mean 300 and standard deviation 5 (may be shown graphically, via calculator normalcdf syntax, in words, in standard notation N(300,5), or via a labeled z-score calculation), (2) specifies the correct event (boundary values 296 and 304, correct direction) or an event consistent with component 1, (3) provides the correct probability 0.5763, or a probability

Solution 6

consistent with components 1 and 2. Partial credit if only 2 of the 3 components are satisfied. Minor errors in statistical notation may be ignored (though considered poor communication under holistic scoring). If the only error is a reversed z-score numerator (300—296 or 300—304), the response is scored partially correct.

Solution 6

(b)(i) Mark scheme

- If the machine is working properly, the sample mean amount of gold $\bar{X}$ (for a sample of n=2 necklaces) has a sampling distribution that is approximately normal with mean 300 mg and standard deviation $5/\sqrt{2} \approx 3.5355$ mg. Then $P(\bar{X} > 303) = P(Z > (303-300)/3.5355) \approx P(Z > 0.8485) \approx 0.198$. NOTE: parts (b)(i) and (b)(ii) are scored TOGETHER as ONE holistic unit in the official scoring guidelines, so this leaf carries this part's full mark. Full credit for the combined part (b) needs components 1, 4, and 5 AND at least one of components 2 or 3: (1) in part (b)(i), indicates use of a normal/approximately-normal distribution and identifies the correct parameter values (mean 300, SD $5/\sqrt{2} \approx 3.5355$) with work shown or a correct formula, (2) in part (b)(i), specifies the correct event (boundary

Solution 6

value and direction: $\bar{X} > 303$), (3) in part (b)(i), provides the correct probability 0.198 (or one consistent with components 1-2), (4) in part (b)(ii), indicates whether observing a sample mean of 303 mg would provide convincing evidence the population mean is not 300 mg, consistent with the response to (b)(i) [graded together with leaf 6(b)(ii)], (5) in part (b)(ii), provides justification based on the probability obtained in (b)(i) or a correctly-computed probability in (b)(ii). An alternate solution using the TOTAL amount of gold on the 2 necklaces (N(600, 7.071), P(total$>$606)) earning full component-1/2/3 credit is also acceptable.

Solution 6

(b)(ii) Mark scheme

- Observing a sample mean amount of 303 mg would NOT provide convincing evidence that the population mean amount of gold being applied by the machine is something other than 300 mg, because the probability of observing a sample mean that differs from 300 mg by 3 mg or more (in either direction) is large - around $0.198(2) = 0.396$ - so a difference this size is not statistically surprising if the machine were working properly. This leaf's credit is bundled with 6(b)(i) in the official scoring guidelines (Part (b) of the SG scores both sub-parts together as one holistic unit), so it carries 0 of this question's marks on its own - but a correct, well-justified 'no' answer, consistent with the probability found in (b)(i), is still expected to complete credit for part (b).

Solution 6

(c)(i) Mark scheme

- Cleo simulates the sampling distribution of the sample range (max−min) for random samples of size n=2 from a normal distribution with standard deviation σ=5 mg (100,000 simulated samples). The resulting sampling distribution of the sample range is skewed to the right. Almost all simulated values of the range fall between 0 mg and about 25-30 mg, and the center of the distribution is about 6 mg. NOTE: parts (c)(i) and (c)(ii) are scored TOGETHER as ONE holistic unit in the official scoring guidelines, so this leaf carries this part's full mark. Full credit for the combined part (c) needs 4 or 5 of the following 5 components: (1) in part (c)(i), describes the shape as positively skewed (skewed right), (2) in part (c)(i), describes the center of the distribution as about 6 mg (any value between 4

Solution 6

and 10 mg is a reasonable center description), (3) in part (c)(i), describes the spread as having most values between 0 mg and 30 mg (or IQR between 5 and 7 mg inclusive), (4) in part (c)(ii), indicates the mean/center of the sample-range distribution increases as the population SD σ increases [graded together with leaf 6(c)(ii)], (5) in part (c)(ii), indicates the variation/spread of the sample-range distribution also increases as σ increases. Partial credit if only 3 of the 5 components are satisfied.

Solution 6

(c)(ii) Mark scheme

- As the value of the population standard deviation σ increases, the variation (spread) in the distribution of the sample range increases, and the mean (center) of the distribution of the sample range also increases. This leaf's credit is bundled with 6(c)(i) in the official scoring guidelines (Part (c) of the SG scores both sub-parts together as one holistic unit), so it carries 0 of this question's marks on its own - but stating BOTH that the center increases AND that the spread increases (as σ increases) is still expected to complete credit for part (c). Comments on skewness are not required here and should be treated as extraneous.

Solution 6

(d)(i) Mark scheme

- No, a sample range of 10 mg is not unusual if the machine is working properly with a standard deviation of 5 mg. Although observing a sample range around 10 mg or greater is much more likely if the population SD is 8 mg or 12 mg, the sampling distribution of the sample range (n=2) for a normal distribution with σ=5 mg shows that 10 mg is not an unusual value - there is about a 20% chance that a random sample of 2 necklaces would yield a range of 10 mg or more when the machine is working properly. NOTE: parts (d)(i) and (d)(ii) are scored TOGETHER as ONE holistic unit in the official scoring guidelines, so this leaf carries this part's full mark. Full credit for the combined part (d) needs 4 or 5 of the following 5 components: (1) in part (d)(i), indicates a sample range of 10 mg

Solution 6

is not unusual, (2) in part (d)(i), justifies this by indicating the $\sigma=5$ sampling-distribution graph shows values of 10 mg or greater occur fairly often, (3) in part (d)(ii), indicates there is not convincing evidence the machine is not working properly [graded together with leaf 6(d)(ii)], (4) in part (d)(ii), justifies this by indicating a sample mean of 303 mg would not be unusual (e.g. it is less than one SD from 300, or consistent with the $\approx$0.198 probability from part (b)), (5) in part (d)(ii), justifies this by indicating a sample range of 10 mg is not unusual, consistent with (d)(i). Partial credit if at least 3 of the 5 components are satisfied.

Solution 6

(d)(ii) Mark scheme

- No, Cleo's sample mean of 303 mg and range of 10 mg do NOT indicate that the machine is not working properly. As noted in part (b)(i), the probability that the sample mean would be equal to or greater than 303 mg when the machine is working properly is almost 20%, so a sample mean of 303 mg is not unusual (it is also less than one standard deviation, $5/\sqrt{2}\approx 3.5355$ mg, away from 300 mg). As indicated in part (d)(i), the probability of a range of 10 mg or greater when the population SD is 5 mg is also about 20%, so it too is not unusual. There is not statistically significant evidence to show the machine is not working properly. This leaf's credit is bundled with 6(d)(i) in the official scoring guidelines (Part (d) of the SG scores both sub-parts together as one holistic unit), so it carries 0 of this

Solution 6

question's marks on its own - but this correct 'no' conclusion, justified by BOTH the mean's and the range's probabilities not being unusual (consistent with (d)(i)), is still expected to complete credit for part (d). Context (amount of gold in necklaces) is not required in parts (a)-(d) but is considered good communication.

Question 6

2019 ▪ Q6

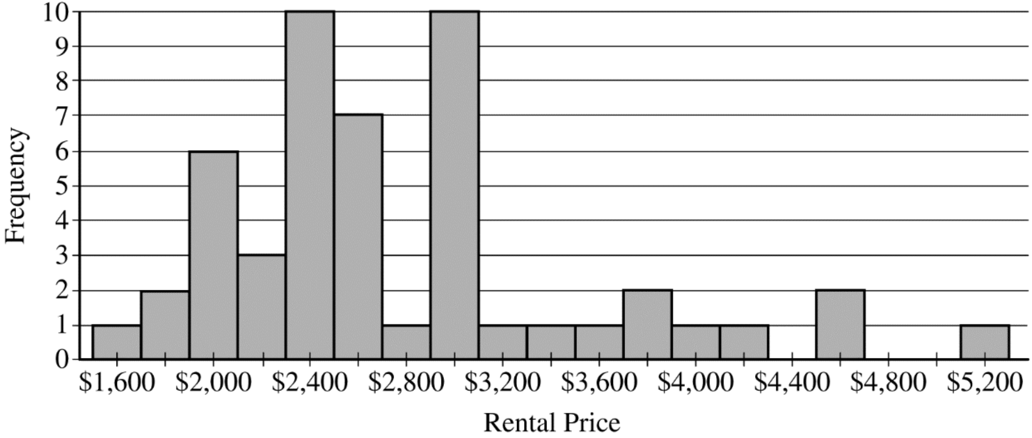
Emma is moving to a large city and is investigating typical monthly rental prices of available one-bedroom apartments. She obtained a random sample of rental prices for 50 one-bedroom apartments taken from a Web site where people voluntarily list available apartments.

(a) Describe the population for which it is appropriate for Emma to generalize the results from her sample.

Question 6

Bootstrap Distribution of Medians					
Median	Frequency	Median	Frequency	Median	Frequency
2,345	1	2,585	1	2,825	247
2,390	13	2,587.5	171	2,837.5	7
2,395	18	2,600	22	2,847.5	1
2,400	56	2,612.5	1,190	2,872.5	317
2,445	4	2,625	174	2,885	10
2,447.5	56	2,672.5	5	2,950	700
2,450	55	2,675	1,924	2,962.5	93
2,475	3	2,687.5	1,341	2,972.5	6
2,495	66	2,700	2,825	2,975	65
2,497.5	136	2,735	35	2,985	12
2,500	1,899	2,747.5	619	2,987.5	1
2,522.5	2	2,750	2	2,995	6
2,525	945	2,795	278	3,000	2
2,550	1,673	2,812.5	16	3,062.5	3

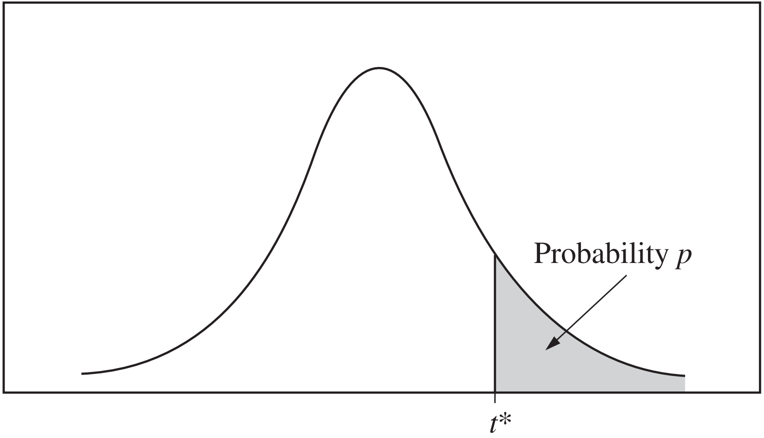
Question 6



Frequency
10
9
8
7
6
5
4
3
2
1
0
$1,600
$2,000
$2,400
$2,800
$3,200
$3,600
$4,000
$4,400
$4,800
$5,200
Rental Price

Question 6

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.



Question 6

2019 ▪ Q6

Emma is moving to a large city and is investigating typical monthly rental prices of available one-bedroom apartments. She obtained a random sample of rental prices for 50 one-bedroom apartments taken from a Web site where people voluntarily list available apartments.

(b) The distribution of the 50 rental prices of the available apartments is shown in the following histogram.

[Histogram titled "Rental Price"; y-axis "Frequency" from 0 to 10; x-axis with tick marks at $1,600, $2,000, $2,400, $2,800, $3,200, $3,600, $4,000, $4,400, $4,800, $5,200.]

Emma wants to estimate the typical rental price of a one-bedroom apartment in the city. Based on the distribution shown, what is a disadvantage of using the mean rather than the median as an estimate of the typical rental price?

Question 6

2019 ▪ Q6

Emma is moving to a large city and is investigating typical monthly rental prices of available one-bedroom apartments. She obtained a random sample of rental prices for 50 one-bedroom apartments taken from a Web site where people voluntarily list available apartments.

(c) Instead of using the sample median as the point estimate for the population mean, Emma wants to explore using the sampling distribution of the sample median for samples of size 50. Emma has one point, her sample median, that she can use to approximate the sampling distribution of the sample medians of size 50. Instead, Emma decides to use a theoretical sampling distribution to approximate the sampling distribution of the sample medians of size 50.

Question 6

Because Emma does not have the resources to develop the theoretical sampling distribution, she estimates the sampling distribution of the sample median using a process called bootstrapping. In the bootstrapping process, a computer program performs the following steps.

- Take a random sample, with replacement, of size 50 from the original sample.
- Calculate and record the median of the sample.
- Repeat the process to obtain a total of 15,000 medians.

Emma ran the bootstrap process, and the following frequency table shows the results of generating 15,000 medians.

Bootstrap Distribution of Medians

Median	Frequency	Median	Frequency	Median	Frequency
2,345	3	2,585	1	2,825	247
2,390	13	2,587.5	171	2,837.5	7
2,395	2	2,600	22	2,847.5	1
2,400	56	2,612.5	1,190	2,872.5	317
2,445	4	2,625	174	2,885	10
2,447.5	56	2,672.5	5	2,950	700
2,450	55	2,675	1,924	2,962.5	93
2,475	3	2,687.5	1,341	2,972.5	6
2,495	66	2,700	2,825	2,975	65

Question 6

The bootstrap distribution provides an approximation of the sampling distribution of the sample median. A confidence interval for the median can be constructed using a percentage of the values in the middle of the bootstrap distribution.

Question 6

2019 ▪ Q6

Emma is moving to a large city and is investigating typical monthly rental prices of available one-bedroom apartments. She obtained a random sample of rental prices for 50 one-bedroom apartments taken from a Web site where people voluntarily list available apartments.

(d)(i) Use the frequency table to find the following.
Value of the 5th percentile.

(d)(ii) Use the frequency table to find the following.
Value of the 95th percentile.

Question 6

2019 ▪ Q6

Emma is moving to a large city and is investigating typical monthly rental prices of available one-bedroom apartments. She obtained a random sample of rental prices for 50 one-bedroom apartments taken from a Web site where people voluntarily list available apartments.

(e) Find the percentage of bootstrap medians in the table that are equal to or between those found in part (d).

Question 6

2019 ▪ Q6

Emma is moving to a large city and is investigating typical monthly rental prices of available one-bedroom apartments. She obtained a random sample of rental prices for 50 one-bedroom apartments taken from a Web site where people voluntarily list available apartments.

(f) Use your values from parts (d) and (e) to construct and interpret a confidence interval for the median rental price.

Solution 6

(a)

Mark scheme

- Because random sampling was used, the results of the sample may be generalized to the population of rental prices for one-bedroom apartments in the city that are listed on this particular website at the time the sample was taken.

Solution 6

(b) Mark scheme

- Because the distribution of the 50 rental prices in the sample is skewed to the right, the sample median provides a better indicator of typical rental prices than the sample mean. Some very large (high) rental prices pull the sample mean up, so the sample mean would overestimate the typical rental price, whereas the sample median would give a more accurate representation of a typical rental price. Must both (1) identify that using the mean overestimates the typical rental price, and (2) link this disadvantage to a feature of the distribution's shape (skewness) evident in the histogram – merely noting the mean is larger than the median is not by itself sufficient.

Solution 6

(c)

Mark scheme

- To determine the sampling distribution of median rental prices for random samples of size 50 from this population, Emma would need to obtain every possible sample of 50 one-bedroom apartments from this population (i.e., from the website's full listing population) and compute the median of each sample. The collection of all of these possible sample medians is the theoretical sampling distribution of the sample median.

Solution 6

(d)(i)

Mark scheme

- $(0.05)(15{,}000)=750$. The 5th percentile is a value $x_{0.05}$ such that at least 750 of the 15,000 bootstrap sample medians in the table are less than or equal to $x_{0.05}$. Cumulating frequencies from the smallest sample median in the table (going down the columns) until first reaching 750 values gives $x_{0.05}=\$2{,}500$.

Solution 6

(d)(ii)

Mark scheme

- $(0.95)(15{,}000)=14{,}250$. Similarly, cumulating frequencies from the smallest value until reaching 14,250 (i.e., leaving 750 values above) gives the 95th percentile $x_{0.95}=\$2{,}950$.

Solution 6

(e)

Mark scheme

- The percentage of the 15,000 bootstrap medians that are equal to or between the part-(d) values $(2,500 \mathit{and} 2{,}950)$ is $\frac{14{,}404}{15{,}000} \times 100\% \approx 96.03\%$. (Any answer using plausible part-(d) values between 2,345 and 3,062.5, computed consistently, is acceptable.)

Solution 6

(f) Mark scheme

- Using the results from parts (d) and (e), an approximate 96 percent confidence interval for the median rental price of all one-bedroom apartments listed on this website for this city is $(2, 500$,2,950). Interpretation: we are approximately 96 percent confident that the median rental price of all one-bedroom apartments listed on this website for this city is between $2, 500 and$2,950. Full credit requires: (1) using $2, 500 and$2,950 (the part-(d) percentile values) as the interval endpoints, (2) stating an approximate 90% or 96% confidence level (both accepted, consistent with part (e)), (3) a correct statement in context that the interval is specifically for the median (not the mean) rental price.