Past-paper walk-through

Combining Random Variables ▪ 4.9

eXercise

Questions in this set

- 2023 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2023 ▪ Q3

. Bath fizzies are mineral tablets that dissolve and create bubbles when added to bathwater. In order to increase sales, the Fizzy Bath Company has produced a new line of bath fizzies that have a cash prize in every bath fizzy. Let the random variable, X, represent the dollar value of the cash prize in a bath fizzy. The probability distribution of X is shown in the table.

Question 3

Probability of cash prize, $P(X = x)$	$P(X = \$1)$	0.2	0.05	0.05	0.01	0.01

(a) Based on the probability distribution of X, answer the following. Show your work.

(i) Calculate the proportion of bath fizzies that contain \$1.

(ii) Calculate the proportion of bath fizzies that contain at least \$10.

Question 3

(b) Based on the probability distribution of X, calculate the probability that a randomly selected bath fizzy contains \$100, given that it contains at least \$10. Show your work.

Question 3

Continue your response to **QUESTION 3** on this page.

(c) Based on the probability distribution of X, calculate and interpret the expected value of the distribution of the cash prize in the bath fizzies. Show your work.

Question 3

(d) The Fizzy Bath Company would like to sell the bath fizzies in France, where the currency is euros. Suppose the conversion rate for dollars to euros is 1 dollar = 0.89 euros. Using your expected value from part (c), calculate the expected value, in euros, of the distribution of the cash prize in the bath fizzies. Show your work.

Question 3

GO ON TO THE NEXT PAGE.

Solution 3

(a)(i) Mark scheme

- Let X be the dollar value of the cash prize in a bath fizzy, with distribution P(X=1)=?, P(X=5)=0.2, P(X=10)=0.05, P(X=20)=0.05, P(X=50)=0.01, P(X=100)=0.01. The proportion of bath fizzies containing $1 is P(X=1) = 1$ - (0.2+0.05+0.05+0.01+0.01) = 1 - 0.32 = 0.68 (a correct percentage such as 68 percent is also acceptable). NOTE: parts (a)(i) and (a)(ii) are scored TOGETHER as ONE holistic unit in the official scoring guidelines (a single essentially-correct/partially-correct/incorrect decision spans both sub-parts), so this leaf carries this part's full mark. Full credit for the combined part (a) needs at least 3 of the following 4 components: (1) correctly calculates the proportion containing

Solution 3

$1 as 0.68, (2) shows correct supporting work for it (i.e., 1 minus the sum of the other probabilities), (
as 0.12 [graded together with leaf 3(a)(ii)], (4) shows correct supporting work for
that. An arithmetic or transcription error can be ignored if correct work is shown.

Solution 3

(a)(ii)

Mark scheme

- The proportion of bath fizzies containing at least $10 is P(X \geq 10) = 0.05+0.05+0.01+0.01 = 0.12$ (a correct percentage such as 12 percent is also acceptable). This leaf's credit is bundled with 3(a)(i) in the official scoring guidelines (Part (a) of the SG scores both sub-parts together as one holistic unit spanning components 1-4), so it carries 0 of this question's marks on its own - but a correct, well-supported answer of 0.12 is still expected to complete credit for part (a). An arithmetic or transcription error can be ignored if correct work is shown.

Solution 3

(b) Mark scheme

- Given a bath fizzy contains at least \$10, the probability it contains \$100 is a conditional probability: $P(X=\$100 \mid X \geq \$10) = P(X=\$100)/P(X \geq \$10) = 0.01/0.12 \approx 0.0833$. Full credit needs both: (1) correctly calculates the requested conditional probability, (2) shows work consistent with the response given to part (a)(ii) - i.e. uses the same value of $P(X \geq \$10)$ as the denominator. A specific conditional-probability notation/statement is not required, but if correctly given should be considered a positive in holistic scoring. An arithmetic or transcription error can be ignored if correct work is shown.

Solution 3

(c) Mark scheme

- The expected value of the distribution of X is $E(X) = 1(0.68) + 5(0.2) + 10(0.05) + 20(0.05) + 50(0.01) + 100(0.01) = 4.68$. *Theexpectedvalueisthemeanofthecashprizesthatresultfromthelongrunofmany, manytr* AND shows appropriate supporting work including at least two of the $x \cdot P(x)$ terms - calculator notation like '1-Var Stats(L1,L2)' does not satisfy the work component) AND at least 2 of: (3) interpretation includes the concept of repeating the selection process over a long period of time, (4) interpretation includes the concept of an average or mean, (5) interpretation includes the context of receiving a cash prize. Partial credit if the response satisfies 2 or 3 of

Solution 3

components 1-4 but not both 1 and 2. An arithmetic or transcription error can be ignored if correct work is shown.

Solution 3

(d) Mark scheme

- The expected value of the distribution of X in euros is $4.68(0.89) \approx 4.17$ euros (using the conversion rate 1 dollar $= 0.89$ euros and the expected value \$4.68 from part (c); alternatively, converting every value of the probability distribution to euros first and recomputing the expected value directly is also acceptable). Full credit needs a correctly-calculated euro value WITH work shown AND units included. A response missing either the work or the units earns only partial credit. A response that does not carry at least one decimal place in the final answer (e.g. rounds to an integer, '4 euros') should be scored no higher than partially correct. An arithmetic or transcription error can be ignored if correct work is shown.