

Exercise walk-through

Combining Random Variables ■ 4.9

eXercise

You must be able to

- find the mean of $aX + bY$ as $a\mu_X + b\mu_Y$
- find the variance of $aX + bY$ for **independent** X, Y as $a^2\sigma_X^2 + b^2\sigma_Y^2$
- describe a **linear transformation** $Y = a + bX$: $\mu_Y = a + b\mu_X$, $\sigma_Y = |b|\sigma_X$

Method — Means add

$$\mu_{aX+bY} = a\mu_X + b\mu_Y \quad (\text{always}).$$

Method — Variances add (if independent)

$$\sigma_{aX+bY}^2 = a^2\sigma_X^2 + b^2\sigma_Y^2 \quad (\text{independent}).$$

Note that **variances**, not standard deviations, add.

Method — Linear transformation

For $Y = a + bX$: $\mu_Y = a + b\mu_X$ and $\sigma_Y = |b|\sigma_X$ (adding a constant does not change the spread).

Practice 2.1 ■ 1 mark

State the mean of $X + Y$ in terms of μ_X and μ_Y .

Solution 2.1

Worked steps

$$\mu_X + \mu_Y \quad \text{B1}.$$

Practice 2.2 ■ 1 mark

If $\mu_X = 5$ and $\mu_Y = 3$, find the mean of $X + Y$.

Solution 2.2

Worked steps

$$5 + 3 = 8 \quad \text{B1}.$$

Practice 2.3 ■ 2 marks

For $Y = 2X$ with $\sigma_X = 4$, find σ_Y .

Solution 2.3

Worked steps

$$\sigma_Y = |2|\sigma_X = 2 \times 4 = 8 \quad \text{M1} \quad \text{A1}.$$

Exam-style 3.1 ■ 1 mark

The mean of $X + Y$ is

A $\mu_X \cdot \mu_Y$

B $\mu_X + \mu_Y$

C $\mu_X - \mu_Y$

D μ_X / μ_Y

Solution 3.1

A $\mu_X \cdot \mu_Y$

B $\mu_X + \mu_Y$



C $\mu_X - \mu_Y$

D μ_X / μ_Y

Worked steps

B.

Exam-style 3.2 ■ 1 mark

For **independent** X and Y , the variance of $X + Y$ is

A $\sigma_X + \sigma_Y$

B $\sigma_X^2 + \sigma_Y^2$

C $\sigma_X \cdot \sigma_Y$

D $\sigma_X^2 - \sigma_Y^2$

Solution 3.2

A $\sigma_X + \sigma_Y$

B $\sigma_X^2 + \sigma_Y^2$



C $\sigma_X \cdot \sigma_Y$

D $\sigma_X^2 - \sigma_Y^2$

Worked steps

B.

Exam-style 3.3 ■ 1 mark

X has $\mu_X = 10$ and $\sigma_X = 2$. Let $Y = 3X$.

(a) Find μ_Y .

(b) Find σ_Y .

Solution 3.3

Worked steps

(a) $\mu_Y = 3 \times 10 = 30$ **B1**. (b) $\sigma_Y = |3| \times 2 = 6$ **M1** **A1**.