Past-paper walk-through

Estimating Probabilities Using Simulation ▪ 4.2

eXercise

Questions in this set

- 2022 Q5
- 2014 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 5

2022 ▪ Q5

Studies have shown that foods rich in compounds known as flavonoids help lower blood pressure. Researchers conducted a study to investigate whether there was a greater reduction in blood pressure for people who consumed dark chocolate, which contains flavonoids, than people who consumed white chocolate, which does not contain flavonoids. Twenty-five healthy adults agreed to participate in the study and add 3.5 ounces of chocolate to their daily diets. Of the 25 participants, 13 were randomly assigned to the dark chocolate group and the rest were assigned to the white chocolate group. All participants had their blood pressure recorded, in millimeters of mercury (mmHg), before adding chocolate to their daily diets and again 30 days after adding chocolate to their daily diets.

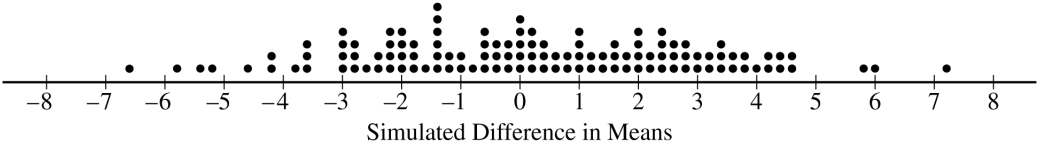
The reduction in blood pressure (before minus after) for each of the participants in the two groups is shown in the dotplots below.

Question 5

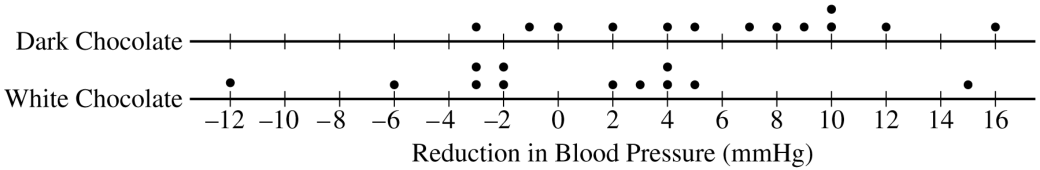
[Dotplots titled with x-axis "Reduction in Blood Pressure (mmHg)" from -12 to 16. "Dark Chocolate" row: approximate dot values at -2, 1, 2, 3, 4, 5, 6, 7, 8, 9, 9, 10 (highest stack), 16. "White Chocolate" row: approximate dot values at -12, -6, -2, -1, 3, 4, 4, 5, 16.]

(a) Determine and compare the medians of the reduction in blood pressure for the two groups.

Question 5



Question 5


Dark Chocolate
White Chocolate
–12 –10 –8 –6 –4 –2 0 2 4 6 8 10 12 14 16
Reduction in Blood Pressure (mmHg)

Question 5

2022 ▪ Q5

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(b) The researchers found the mean reduction in blood pressure for those who consumed dark chocolate is $\bar{x}_{dark} = 6.08$ mmHg and the mean reduction in blood pressure for those who consumed white chocolate is $\bar{x}_{white} = 0.42$ mmHg.

One researcher indicated that because the difference in sample means of 5.66 mmHg is greater than 0 there is convincing statistical evidence to conclude that the population mean blood pressure reduction for those who consume dark chocolate is greater than for those who consume white chocolate. Why might the researcher’s conclusion, based only on the difference in sample means of 5.66 mmHg, not necessarily be true?

Question 5

2022 ▪ Q5

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(c) A simulation was conducted to investigate whether there is a greater reduction of blood pressure for those who consume dark chocolate than for those who consume white chocolate. The simulation was conducted under the assumption that no difference exists. The results of 120 trials of the simulation are shown in the following dotplot.

[Dotplot titled with x-axis "Simulated Difference in Means" from -8 to 8; roughly symmetric, mound-shaped distribution of 120 dots centered near 0, tallest stacks between -2 and 3, tapering to isolated dots near -8, -7, -6, -5, 6, 7.]

Use the results of the simulation to determine whether the results from the 25 participants in the study provide convincing statistical evidence, at a 5 percent level of

Question 5

significance, that adding dark chocolate to a daily diet will result in a greater reduction in blood pressure, on average, than adding white chocolate to a daily diet. Justify your answer.

Solution 5

(a)

Mark scheme

- The sample median reduction in blood pressure for those eating dark chocolate is 7 mmHg, which is greater than the sample median reduction for those eating white chocolate, about 0 mmHg. Full credit needs all three: (1) the medians are correctly determined (dark chocolate median $= 7$ mmHg; white chocolate median between -2 and 2 mmHg, inclusive), (2) the computed medians are correctly compared, (3) context is included (the response variable, reduction in blood pressure, and the two treatment groups, dark and white chocolate).

Solution 5

(b) Mark scheme

- The researcher's conclusion may not necessarily be true because looking at the difference in sample means alone does not account for the variability in the sampling distribution of the difference in sample means. A different random assignment of subjects to dark and white chocolate could result in a sample mean reduction for dark chocolate that is smaller than for white chocolate. The variability in the sampling distribution of potential differences in sample means must be considered when concluding convincing statistical evidence; an inference procedure can assess the likelihood that the observed difference in sample means (5.66 mmHg) occurred by random chance if the population means are actually equal. Full credit needs at least two of the following three: (1) indicates the

Solution 5

researchers failed to consider the variability in the sampling distribution of differences in sample means (or that this should be considered), or states a different random assignment could produce different sample means, (2) explains that a difference of 5.66 mmHg could have occurred by random chance even if the population means are equal, (3) indicates that an inference procedure is needed. (A response may also earn full credit by instead correctly noting that no population conclusion can be drawn at all, because the sample was drawn only from the subpopulation of healthy adults.)

Solution 5

(c) Mark scheme

- The observed value of the sample statistic $\bar{x}_{dark} - \bar{x}_{white}$ is 5.66 mmHg. The simulation (under the assumption of no true difference) shows that a difference of 5.66 mmHg or larger occurred in only 3 of the 120 trials, so the (simulation-based) *p*-value is approximately $\frac{3}{120} = 0.025$. Because this *p*-value is less than 0.05, there is convincing statistical evidence that adding dark chocolate to the diet results in a greater mean reduction in blood pressure than adding white chocolate, for people similar to those in the study. (The hypotheses themselves are not required since they are given in the stem.) Full credit needs all four components: (1) calculates the correct *p*-value of 0.025, (2) provides

Solution 5

supporting work from the simulation for the calculation (e.g. noting only 3 of the 120 trials met or exceeded 5.66 mmHg), (3) provides a correct conclusion in context about whether there is convincing evidence that dark chocolate gives a greater mean blood-pressure reduction than white chocolate, (4) justifies the conclusion by directly comparing the calculated p-value to 0.05.

Question 2

2014 ▪ Q2

Nine sales representatives, 6 men and 3 women, at a small company wanted to attend a national convention. There were only enough travel funds to send 3 people. The manager selected 3 people to attend and stated that the people were selected at random. The 3 people selected were women. There were concerns that no men were selected to attend the convention.

(a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.

Question 2

Each trial in the simulation consists of rolling three fair, six-sided dice, one die for each of the convention attendees. For each die, rolling a 1, 2, 3, or 4 represents selecting a man; rolling a 5 or 6 represents selecting a woman. After 1,000 trials, the number of times the dice indicate selecting 3 women is recorded.

Question 2

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(b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

Question 2

2014 ▪ Q2

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(c) An alternative to calculating the exact probability is to conduct a simulation to estimate the probability. A proposed simulation process is described below.

Each trial in the simulation consists of rolling three fair, six-sided dice, one die for each of the convention attendees. For each die, rolling a 1, 2, 3, or 4 represents selecting a man; rolling a 5 or 6 represents selecting a woman. After 1,000 trials, the number of times the dice indicate selecting 3 women is recorded.

Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

Solution 2

(a) Mark scheme

- (Q2 is graded holistically across parts (a)-(c) via a combination table into an overall 0-4 score; this leaf's marks approximate its share of the 4-point total.) Using the multiplication rule without replacement: P(all 3 selected are women) $=$ P(1st is woman) $\times$ P(2nd is woman $|$ 1st woman) $\times$ P(3rd is woman $|$ first two women) $= (3/9)(2/8)(1/7) \approx 0.012$. Full credit needs the probability correctly computed with the computation shown. Partial credit for any ONE of: stating $1/84$ (0.012 or 0.011 accepted) with no work shown; correctly showing how the probability should be computed but not carrying the computation through correctly; correctly computing (with work) only the numerator or only the denominator of the correct answer (e.g. $(1/9)(1/8)(1/7) \approx 0.002$, or

Solution 2

$(3/9)(3/9)(3/9) \approx 0.008$, or $(3/9)(3/8)(3/7) \approx 0.054$); or mistakenly assuming independence and computing, with work shown, the binomial-style probability $(1/3)(1/3)(1/3) = 1/27 \approx 0.037$.

Solution 2

(b) Mark scheme

- Yes, the probability from part (a) does provide a reason to doubt the manager's claim that the selections were made at random: there is only about a 1.2% chance that random selection would have resulted in three women being chosen, which is small enough to cast doubt on the claim that the selections were made at random. Full credit needs the response to state that the part (a) probability is small (or insufficiently small), make an appropriate decision consistent with that small probability, and do so in the context of this situation. Partial credit for otherwise meeting full credit but without context, or for stating a significance level and making an appropriate contextual decision without explicitly comparing the probability to that level, or for otherwise meeting full credit but not explicitly

Solution 2

deciding whether there is reason to doubt the manager's claim (e.g. only restating that the probability is very small). Notes: a response that says the small probability 'proves' the manager did not select at random, or that clearly reinterprets the part (a) probability as the probability the manager DID select at random, should be scored one level down from what it otherwise would earn (E$\rightarrow$P, P$\rightarrow$I); a response with a decision inconsistent with its own justification, or one that argues the selection was not impossible so there is no reason to doubt the claim, is incorrect.

Solution 2

(c) Mark scheme

- No, the proposed dice simulation does not correctly simulate the random selection of three women from the group of nine (six men, three women). The random selection of three people among nine is done WITHOUT replacement, but the three dice rolls in any given trial are independent of one another, so the simulation actually samples WITH replacement (equivalently: the probability of rolling a 5 or 6 stays fixed at $1/3$ on every die, whereas the true probability of selecting a woman changes as people are removed without replacement). An equally acceptable route: compute the probability a simulation trial indicates 3 women as $(1/3)(1/3)(1/3) \approx 0.037$ and state this differs from the 0.012 probability found in part (a). Full credit needs the response to answer 'No' AND state that the dice

Solution 2

outcomes are independent AND state that the genders of the selected attendees are dependent (equivalent phrasings: the probability of selecting a woman changes after each selection; the people are sampled without replacement; pointing out the sample of 3 is more than 10% of the population of 9 also satisfies the dependence component) — OR answers 'No' and computes the $(1/3)\hat{}3 \approx 0.037$ trial probability and states it differs from part (a)'s. Partial credit for answering 'No' and correctly stating only ONE of independence-of-dice or dependence-of-genders (not both), or for otherwise meeting full credit but with poor communication that doesn't clearly connect the simulation description to the actual sampling process (e.g. describing the simulation as 'with replacement' then describing the real selection as 'it isn't possible to select the same person twice' without tying the two together).