Past-paper walk-through

Introducing Statistics: Random and Non-Random Patterns? ▪ 4.1

eXercise

Questions in this set

- 2014 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

2014 ▪ Q2

Nine sales representatives, 6 men and 3 women, at a small company wanted to attend a national convention. There were only enough travel funds to send 3 people. The manager selected 3 people to attend and stated that the people were selected at random. The 3 people selected were women. There were concerns that no men were selected to attend the convention.

(a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.

Question 2

Each trial in the simulation consists of rolling three fair, six-sided dice, one die for each of the convention attendees. For each die, rolling a 1, 2, 3, or 4 represents selecting a man; rolling a 5 or 6 represents selecting a woman. After 1,000 trials, the number of times the dice indicate selecting 3 women is recorded.

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(b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

Question 2

2014 ▪ Q2

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(c) An alternative to calculating the exact probability is to conduct a simulation to estimate the probability. A proposed simulation process is described below.

Each trial in the simulation consists of rolling three fair, six-sided dice, one die for each of the convention attendees. For each die, rolling a 1, 2, 3, or 4 represents selecting a man; rolling a 5 or 6 represents selecting a woman. After 1,000 trials, the number of times the dice indicate selecting 3 women is recorded.

Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

Solution 2

(a) Mark scheme

- (Q2 is graded holistically across parts (a)-(c) via a combination table into an overall 0-4 score; this leaf's marks approximate its share of the 4-point total.) Using the multiplication rule without replacement: P(all 3 selected are women) $=$ P(1st is woman) $\times$ P(2nd is woman $|$ 1st woman) $\times$ P(3rd is woman $|$ first two women) $= (3/9)(2/8)(1/7) \approx 0.012$. Full credit needs the probability correctly computed with the computation shown. Partial credit for any ONE of: stating $1/84$ (0.012 or 0.011 accepted) with no work shown; correctly showing how the probability should be computed but not carrying the computation through correctly; correctly computing (with work) only the numerator or only the denominator of the correct answer (e.g. $(1/9)(1/8)(1/7) \approx 0.002$, or

Solution 2

$(3/9)(3/9)(3/9) \approx 0.008$, or $(3/9)(3/8)(3/7) \approx 0.054$); or mistakenly assuming independence and computing, with work shown, the binomial-style probability $(1/3)(1/3)(1/3) = 1/27 \approx 0.037$.

Solution 2

(b) Mark scheme

- Yes, the probability from part (a) does provide a reason to doubt the manager's claim that the selections were made at random: there is only about a 1.2% chance that random selection would have resulted in three women being chosen, which is small enough to cast doubt on the claim that the selections were made at random. Full credit needs the response to state that the part (a) probability is small (or insufficiently small), make an appropriate decision consistent with that small probability, and do so in the context of this situation. Partial credit for otherwise meeting full credit but without context, or for stating a significance level and making an appropriate contextual decision without explicitly comparing the probability to that level, or for otherwise meeting full credit but not explicitly

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deciding whether there is reason to doubt the manager's claim (e.g. only restating that the probability is very small). Notes: a response that says the small probability 'proves' the manager did not select at random, or that clearly reinterprets the part (a) probability as the probability the manager DID select at random, should be scored one level down from what it otherwise would earn (E$\rightarrow$P, P$\rightarrow$I); a response with a decision inconsistent with its own justification, or one that argues the selection was not impossible so there is no reason to doubt the claim, is incorrect.

Solution 2

(c) Mark scheme

- No, the proposed dice simulation does not correctly simulate the random selection of three women from the group of nine (six men, three women). The random selection of three people among nine is done WITHOUT replacement, but the three dice rolls in any given trial are independent of one another, so the simulation actually samples WITH replacement (equivalently: the probability of rolling a 5 or 6 stays fixed at $1/3$ on every die, whereas the true probability of selecting a woman changes as people are removed without replacement). An equally acceptable route: compute the probability a simulation trial indicates 3 women as $(1/3)(1/3)(1/3) \approx 0.037$ and state this differs from the 0.012 probability found in part (a). Full credit needs the response to answer 'No' AND state that the dice

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outcomes are independent AND state that the genders of the selected attendees are dependent (equivalent phrasings: the probability of selecting a woman changes after each selection; the people are sampled without replacement; pointing out the sample of 3 is more than 10% of the population of 9 also satisfies the dependence component) — OR answers 'No' and computes the $(1/3)\hat{}3 \approx 0.037$ trial probability and states it differs from part (a)'s. Partial credit for answering 'No' and correctly stating only ONE of independence-of-dice or dependence-of-genders (not both), or for otherwise meeting full credit but with poor communication that doesn't clearly connect the simulation description to the actual sampling process (e.g. describing the simulation as 'with replacement' then describing the real selection as 'it isn't possible to select the same person twice' without tying the two together).