Past-paper walk-through

The Geometric Distribution ▪ 4.12

eXercise

Questions in this set

- 2026 Q3
- 2024 Q4
- 2016 Q4

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2026 ▪ Q3

A certain sports team has its team song performed by a different musician at the beginning of each of its games. The time it takes for the team song to be performed by different musicians can be modeled using a normal distribution with mean $\mu = 109$ seconds and standard deviation $\sigma = 16$ seconds. All performances of the team song are independent.

(a) What is the probability that a randomly selected performance of the team song will take longer than 120 seconds? Show your work.

(b) Ten performances of the team song will be randomly selected. Let the random variable X represent the number of games at which the performance of the team song takes longer than 120 seconds. Find $P(X \geq 3)$. Show your work.

Question 3

(c)(i) Ben will attend all of the games for this sports team. Let the random variable Y represent the number of games Ben will attend until a performance of the team song takes longer than 120 seconds.
Calculate the mean of Y. Show your work.
(c)(ii) Calculate the standard deviation of Y. Show your work.
(d) Interpret the standard deviation calculated in part C (ii) in context.

Question 4

2024 ▪ Q4

In an online game, players move through a virtual world collecting geodes, a type of hollow rock. When broken open, these geodes contain crystals of different colors that are useful in the game. A red crystal is the most useful crystal in the game. The color of the crystal in each geode is independent and the probability that a geode contains a red crystal is 0.08.

(a)(i) Sarah, a player, will collect and open geodes until a red crystal is found. Calculate the mean of the distribution of the number of geodes Sarah will open until a red crystal is found. Show your work.

(a)(ii) Calculate the standard deviation of the distribution of the number of geodes Sarah will open until a red crystal is found. Show your work.

Question 4

2024 ▪ Q4

In an online game, players move through a virtual world collecting geodes, a type of hollow rock. When broken open, these geodes contain crystals of different colors that are useful in the game. A red crystal is the most useful crystal in the game. The color of the crystal in each geode is independent and the probability that a geode contains a red crystal is 0.08.

(b) Another player, Conrad, decides to play the game and will stop opening geodes after finding a red crystal or when 4 geodes have been opened, whichever comes first. Let $Y =$ the number of geodes Conrad will open. The table shows the partially completed probability distribution for the random variable Y.

Question 4

Number of geodes Conrad will open, y	**1**	**2**	**3**	**4**
Probability, $P(Y = y)$	0.08	0.0736		

(b)(i) Calculate $P(Y = 3)$. Show your work.
(b)(ii) Calculate $P(Y = 4)$. Show your work.

Question 4

2024 ▪ Q4

In an online game, players move through a virtual world collecting geodes, a type of hollow rock. When broken open, these geodes contain crystals of different colors that are useful in the game. A red crystal is the most useful crystal in the game. The color of the crystal in each geode is independent and the probability that a geode contains a red crystal is 0.08.

(c)(i) Consider the table and your results from part (b).
Calculate the mean of the distribution of the number of geodes Conrad will open.
Show your work.

(c)(ii) Interpret the mean of the distribution of the number of geodes Conrad will open, which was calculated in part (c-i).

Solution 4

(a)(i)

Mark scheme

- Let $G =$ the number of geodes Sarah opens until a red crystal is found; G follows a geometric distribution with $p = 0.08$. $\mu = E(G) = \dfrac{1}{0.08} \approx 12.5$ geodes. Full credit needs the correct mean (12.5; rounding to 12 or 13 does NOT satisfy this) with supporting work showing the formula $1/p$. An arithmetic/transcription error can be ignored if correct work is shown.

Solution 4

(a)(ii)

Mark scheme

- $\sigma_G = \dfrac{\sqrt{1-0.08}}{0.08} \approx 11.99$ geodes. Full credit needs the correct standard deviation (a response of 12 geodes also satisfies this) with supporting work showing the geometric-distribution SD formula $\sqrt{1-p}\,/\,p$.

Solution 4

(b)

Mark scheme

- No independent response is required here – this leaf is context introducing the random variable $Y =$ the number of geodes Conrad opens (he stops at a red crystal or after 4 geodes, whichever comes first) and the partially completed probability table, with $P(Y = 1) = 0.08$ and $P(Y = 2) = 0.0736$ already given. $P(Y = 3)$ and $P(Y = 4)$ are computed in parts (b-i) and (b-ii).

Solution 4

(b)(i)

Mark scheme

- $P(Y = 3) = (0.92)^2(0.08) \approx 0.067712$. Full credit needs the correct probability with supporting work – either the probability formula $(1 - p)^2 p$ / $(0.92)^2(0.08)$, or correct calculator syntax with the parameter identified, e.g. `geompdf(p=0.08, x=3)`.

Solution 4

(b)(ii)

Mark scheme

- $P(Y = 4) = 1 - P(Y = 1 \text{ or } 2 \text{ or } 3) = 1 - (0.08 + 0.0736 + 0.067712) \approx 0.778688$. (Equivalently, $P(Y = 4) = (0.92)^4 + (0.92)^3(0.08) \approx 0.778688$, since Conrad opens a 4th geode only if none of the first three is red, and then it is red or not.) Full credit needs the correct probability, consistent with the value found in part (b-i), with supporting work (a probability formula or correct calculator syntax with parameters identified, e.g. `1-geomcdf(p=0.08, x=3)` or `binompdf(n=4, p=0.08, x=0)+geompdf(p=0.08, x=4)`).

Solution 4

(c)(i)

Mark scheme

- $\mu = E(Y) = (1)(0.08) + (2)(0.0736) + (3)(0.067712) + (4)(0.778688) \approx 0.08 + 0.1472 + 0.2031 + 3.1148 \approx 3.545$ geodes. Full credit needs the correct mean, consistent with the probability distribution in part (b), with supporting work showing at least two of the summed terms (e.g. $1(0.08) + 2(0.0736) + \ldots$); calculator notation alone, such as 1-VAR STATS(L1,L2), does NOT satisfy the supporting-work requirement. An arithmetic/transcription error can be ignored if correct work is shown.

Solution 4

(c)(ii)

Mark scheme

- The mean of 3.545 geodes is the average number of geodes that result from a long run of many, many trials of opening randomly selected geodes and counting the number opened until either a red crystal is found or the fourth geode is opened. Full credit needs the interpretation to include: (1) the concept of repeating the selection process over a long period of time / many trials, (2) the concept of an average or mean, (3) context – the number of geodes opened. The numerical value of the mean is not required to satisfy these three components.

Question 4

2016 ▪ Q4

A company manufactures model rockets that require igniters to launch. Once an igniter is used to launch a rocket, the igniter cannot be reused. Sometimes an igniter fails to operate correctly, and the rocket does not launch. The company estimates that the overall failure rate, defined as the percent of all igniters that fail to operate correctly, is 15 percent.

A company engineer develops a new igniter, called the super igniter, with the intent of lowering the failure rate. To test the performance of the super igniters, the engineer uses the following process.

Step 1: One super igniter is selected at random and used in a rocket.

Step 2: If the rocket launches, another super igniter is selected at random and used in a rocket.

Question 4

Step 2 is repeated until the process stops. The process stops when a super igniter fails to operate correctly or 32 super igniters have successfully launched rockets, whichever comes first. Assume that super igniter failures are independent.

(a) If the failure rate of the super igniters is 15 percent, what is the probability that the first 30 super igniters selected using the testing process successfully launch rockets?

(b) Given that the first 30 super igniters successfully launch rockets, what is the probability that the first failure occurs on the thirty-first or the thirty-second super igniter tested if the failure rate of the super igniters is 15 percent?

(c) Given that the first 30 super igniters successfully launch rockets, is it reasonable to believe that the failure rate of the super igniters is less than 15 percent? Explain.

Solution 4

(a)

Mark scheme

- If the failure rate is 15

Solution 4

(b)

Mark scheme

- Given the first 30 succeed, the probability the first failure occurs on the 31st trial is 0.15, and the probability it occurs on the 32nd trial (31st succeeds, 32nd fails) is $(0.85)(0.15) = 0.1275$. So $P(\text{first failure on the } 31^{st} \text{ or } 32^{nd}) = 0.15 + 0.1275 = 0.2775$. (This is equivalent to asking for the probability the first failure occurs on the first or second of two new trials, since the process is memoryless.) Full credit requires both the correct probability AND correct justification.

Solution 4

(c) Mark scheme

- Yes, it is reasonable to believe the failure rate of the super igniters is less than 15