

Exercise walk-through

Residuals ■ 2.7

eXercise

You must be able to

- calculate a **residual** 残差 as $y - \hat{y}$ (observed minus predicted)
- construct and interpret a **residual plot** 残差图
- use a residual plot to judge whether a **linear model** is appropriate

Method — Residuals

$$\text{residual} = y - \hat{y} = \text{observed} - \text{predicted}.$$

A **positive** residual means the point lies **above** the line.

Method — Residual plots

Plot the residuals against x (or $\hat{y}$):

- **random scatter** with no pattern $\rightarrow$ the linear model is appropriate;
- a **clear pattern** (e.g. a curve) $\rightarrow$ a linear model is **not** appropriate.

Practice 2.1 ■ 1 mark

Write the formula for a residual.

Solution 2.1

Worked steps

$$\text{residual} = y - \hat{y} \quad \text{B1}.$$

Practice 2.2 ■ 2 marks

An observed value is 18 and the model predicts 15. Find the residual.

Solution 2.2

Worked steps

$$18 - 15 = 3 \quad \text{M1} \quad \text{A1}.$$

Practice 2.3 ■ 1 mark

State what a residual plot with a clear pattern suggests.

Solution 2.3

Method: Residual plots

Worked steps

a linear model is not appropriate for the data **B1**.

Exam-style 3.1 ■ 1 mark

A residual equals

A $\hat{y} - y$

B $y - \hat{y}$

C $y + \hat{y}$

D $y \times \hat{y}$

Solution 3.1

A $\hat{y} - y$

B $y - \hat{y}$ 

C $y + \hat{y}$

D $y \times \hat{y}$

Worked steps

B.

Exam-style 3.2 ■ 1 mark

A residual plot showing random scatter suggests the linear model is

A inappropriate

B appropriate

C curved

D wrong

Solution 3.2

Method: Residual plots

A inappropriate

B appropriate 

C curved

D wrong

Worked steps

B.

Exam-style 3.3 ■ 2 marks

A prediction is $\hat{y} = 40$, but the actual value is $y = 46$.

- (a) Find the residual.
- (b) State whether the point is above or below the line.

Solution 3.3

Method: Residuals

Worked steps

(a) $46 - 40 = 6$ **M1** **A1**. (b) above the line (positive residual) **B1**.