

Past-paper walk-through

Linear Regression Models ■ 2.6

eXercise

Questions in this set

- 2026 Q6
- 2023 Q5
- 2022 Q1
- 2021 Q6
- 2016 Q6
- 2015 Q5

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- **//** separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 6

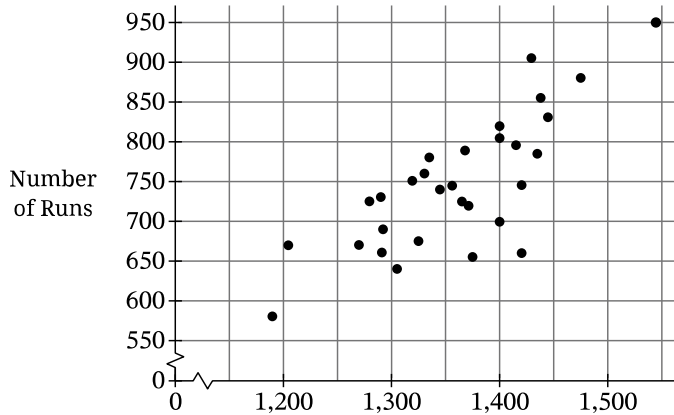
2026 ■ Q6

The following information applies to parts A and B.

6. To win a game, a baseball team needs to score runs. To score runs, players need to get on base. The primary way players get on base is by hitting the ball. Figure 1 shows the relationship between the number of hits and the number of runs for 30 randomly selected professional baseball teams.

Figure 1. Scatterplot of Number of Runs by Number of Hits

Question 6



Question 6

Number of Hits

Part A

Consider the scatterplot in Figure 1.

- Describe the relationship between the number of hits and the number of runs for these baseball teams, in context.
- Based on the data from the sample of 30 teams, an equation of the least-squares regression line model for predicting the number of runs from the number of hits is as follows.

$$\text{Predicted number of runs} = -372.2 + 0.823(\text{number of hits})$$

Question 6

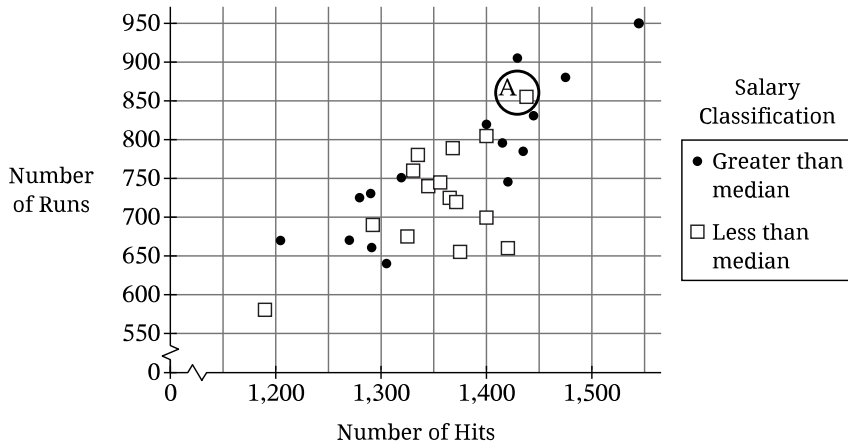
Using the given regression equation, calculate the point estimate for the predicted number of runs a team would score if they were to achieve their goal of 1,250 hits. Show your work.

Part B

Each team has a total salary equal to the sum of the salaries of all players on the team. The median total salary for the sample of 30 teams is \$160 million. In Figure 2, points with dots represent teams with a total salary greater than the median, and points with squares represent teams with a total salary less than the median.

Figure 2. Scatterplot of Number of Runs by Number of Hits and Salary Classification

Question 6



Question 6

Consider the scatterplot in Figure 2.

- i. Compare the team represented by the point that is circled and labeled A with the other teams that have the same total salary classification.
- ii. For each salary classification, consider the linear relationship between the number of hits and the number of runs. For teams with a total salary greater than the median, is the strength of the linear relationship stronger than, weaker than, or similar to the strength of the linear relationship for teams with a total salary less than the median? Explain.

Part C

Different types of intervals can be used when working with linear regression models. One type of interval is a confidence interval for the slope of the least-squares regression line. Two other types of intervals in the context of this problem are as follows:

Question 6

- A confidence interval that estimates the mean number of runs for all teams with a specific number of hits.
- A prediction interval that predicts the number of runs for a single team with a specific number of hits.

All three intervals use the same basic formula:

$$\text{Point Estimate} \pm t^* (\text{standard error}),$$

where critical value t^* comes from a t -distribution that has $n - 2$ degrees of freedom. The same critical value is used when finding each of these intervals with the same level of confidence.

Consider the point estimate for the predicted number of runs found in part A (ii) for the team whose goal is to achieve 1,250 hits next year. Recall that this value was calculated using the linear regression model based on the data from the sample of 30 teams.

Question 6

- i. What is the critical value for a 95% confidence level that would be used for the confidence interval for the mean number of runs as well as for the prediction interval for the number of runs? Indicate the answer to two decimal places.
- ii. The standard error for the confidence interval for the mean number of runs is 17.48. Assuming the conditions for inference are met, calculate the 95% confidence interval for the mean number of runs for all teams with 1,250 hits. Show your work.
- iii. The standard error for the prediction interval for the number of runs is 56.78. Assuming the conditions for inference are met, calculate the 95% prediction interval for the number of runs for a single team with 1,250 hits. Show your work.

Part D

- i. Would a distribution of sample means be expected to have more variability or less variability than a distribution of individual observations? Explain.

Question 6

- ii. The standard error used in the confidence interval that estimates the mean number of runs for all teams with x hits can be found using the following formula.

$$\sqrt{s^2 \left(\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum (x_i - \bar{x})^2} \right)}$$

The standard error used in the prediction interval that predicts the number of runs for a single team with x hits can be found using the following formula.

$$\sqrt{s^2 + s^2 \left(\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum (x_i - \bar{x})^2} \right)}$$

In both standard error formulas, s is the standard deviation of the residuals, n is the sample size, and $\bar{x}$ is the sample mean number of hits. Based on the answer from part D (i) and the standard error formulas for the confidence interval and the prediction interval, explain why the prediction interval calculated in part C (iii) is wider than the confidence interval calculated in part C (ii).

STOP
END OF EXAM

Question 5

2023 ■ Q5

Wildlife biologists are interested in the health of tule elk, a species of deer found in California. An important measurement of tule elk health is their weight. The weight of a tule elk is difficult to measure in the wild. However, chest circumference, which is believed to be related to the weight of a tule elk, can easily be measured from a safe distance using a harmless laser. A study was done to investigate whether chest circumference, in centimeters (cm), could be used to accurately estimate the weight, in kilograms (kg), of male tule elk. For the study, wildlife biologists captured 30 male tule elk, measured their chest circumference and weight, and then released the elk. The data for the 30 male tule elk are shown in the scatterplot.

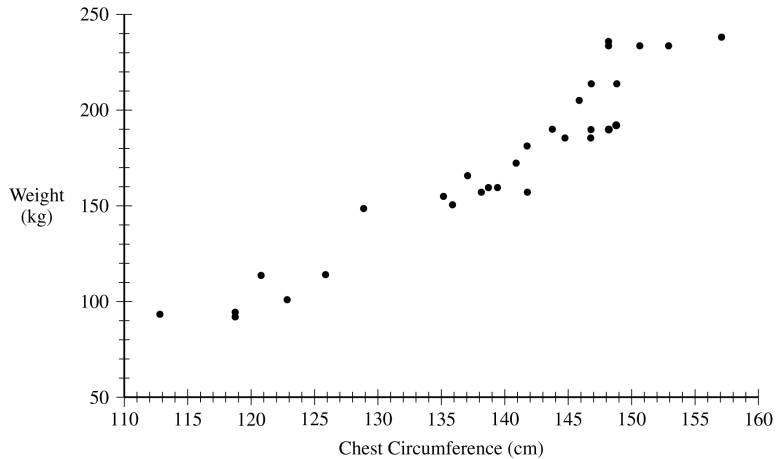
[Scatterplot titled with axes “Chest Circumference (cm)” (x-axis, 110 to 160) and “Weight (kg)” (y-axis, 50 to 250). The points show a strong, positive, roughly linear association: e.g. points near (113,90), (120,93), (121,98), (125,103), (130,145),

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(133,148), (135,150), (137,155), (138,160), (140,163), (141,175), (142,178), (143,180), (144,183), (145,204), (146,207), (147,210), (148,213), (149,233), (150,236), (152,238), (153,240), (155,241), continuing up to about (156,241), with the cloud of points trending upward from lower-left to upper-right.]

(a) Describe the relationship between chest circumference and weight of male tule elk in context.

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2023 ■ Q5

Wildlife biologists are interested in the health of tule elk, a species of deer found in California. An important measurement of tule elk health is their weight. The weight of a tule elk is difficult to measure in the wild. However, chest circumference, which is believed to be related to the weight of a tule elk, can easily be measured from a safe distance using a harmless laser. A study was done to investigate whether chest circumference, in centimeters (cm), could be used to accurately estimate the weight, in kilograms (kg), of male tule elk. For the study, wildlife biologists captured 30 male tule elk, measured their chest circumference and weight, and then released the elk. The data for the 30 male tule elk are shown in the scatterplot.

[Scatterplot titled with axes “Chest Circumference (cm)” (x-axis, 110 to 160) and “Weight (kg)” (y-axis, 50 to 250). The points show a strong, positive, roughly linear association: e.g. points near (113,90), (120,93), (121,98), (125,103), (130,145), (133,148), (135,150), (137,155), (138,160), (140,163), (141,175), (142,178), (143,180), (144,183), (145,204), (146,207), (147,210), (148,213), (149,233), (150,236), (152,238), (153,240), (155,241),

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continuing up to about (156,241), with the cloud of points trending upward from lower-left to upper-right.]

(b)(i) Following is the equation of the least-squares regression line relating chest circumference and weight for male tule elk.

$$\text{predicted weight} = -350.3 + 3.7455(\text{chest circumference})$$

The weight of one male tule elk with a chest circumference of 145.9 cm is 204.3 kg. Using the equation of the least-squares regression line, calculate the predicted weight for this male tule elk. Show your work.

(b)(ii) Calculate the residual for this male tule elk. Show your work.

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2023 ■ Q5

Wildlife biologists are interested in the health of tule elk, a species of deer found in California. An important measurement of tule elk health is their weight. The weight of a tule elk is difficult to measure in the wild. However, chest circumference, which is believed to be related to the weight of a tule elk, can easily be measured from a safe distance using a harmless laser. A study was done to investigate whether chest circumference, in centimeters (cm), could be used to accurately estimate the weight, in kilograms (kg), of male tule elk. For the study, wildlife biologists captured 30 male tule elk, measured their chest circumference and weight, and then released the elk. The data for the 30 male tule elk are shown in the scatterplot.

[Scatterplot titled with axes “Chest Circumference (cm)” (x-axis, 110 to 160) and “Weight (kg)” (y-axis, 50 to 250). The points show a strong, positive, roughly linear association: e.g. points near (113,90), (120,93), (121,98), (125,103), (130,145), (133,148), (135,150), (137,155), (138,160), (140,163), (141,175), (142,178), (143,180), (144,183), (145,204), (146,207), (147,210), (148,213), (149,233), (150,236), (152,238), (153,240), (155,241),

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continuing up to about (156,241), with the cloud of points trending upward from lower-left to upper-right.]

(c) The equation of the least-squares regression line relating chest circumference and weight for male tule elk is repeated here.

$$\text{predicted weight} = -350.3 + 3.7455(\text{chest circumference})$$

Interpret the slope of the least-squares regression line in context.

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2023 ■ Q5

Wildlife biologists are interested in the health of tule elk, a species of deer found in California. An important measurement of tule elk health is their weight. The weight of a tule elk is difficult to measure in the wild. However, chest circumference, which is believed to be related to the weight of a tule elk, can easily be measured from a safe distance using a harmless laser. A study was done to investigate whether chest circumference, in centimeters (cm), could be used to accurately estimate the weight, in kilograms (kg), of male tule elk. For the study, wildlife biologists captured 30 male tule elk, measured their chest circumference and weight, and then released the elk. The data for the 30 male tule elk are shown in the scatterplot.

[Scatterplot titled with axes “Chest Circumference (cm)” (x-axis, 110 to 160) and “Weight (kg)” (y-axis, 50 to 250). The points show a strong, positive, roughly linear association: e.g. points near (113,90), (120,93), (121,98), (125,103), (130,145), (133,148), (135,150), (137,155), (138,160), (140,163), (141,175), (142,178), (143,180), (144,183), (145,204), (146,207), (147,210), (148,213), (149,233), (150,236), (152,238), (153,240), (155,241),

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continuing up to about (156,241), with the cloud of points trending upward from lower-left to upper-right.]

(d)(i) The sambar, another species of deer, is similar in size to the tule elk. The slope of the population regression line relating chest circumference and weight for all male sambars is 4.5 kilograms per centimeter. A wildlife biologist wants to determine whether the slope of the population regression line for male tule elk is different than that for male sambars. Let β represent the slope of the population regression line for male tule elk. The wildlife biologist conducted a test of the following hypotheses using the sample of 30 tule elk.

$$H_0 : \beta = 4.5$$

$$H_a : \beta \neq 4.5$$

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The test statistic was calculated to be 3.408. Assume all conditions for inference were met.

Determine the p -value of the test.

(d)(ii) At a significance level of $\alpha = 0.05$, what conclusion should the wildlife biologist make regarding the slope of the population regression line for male tule elk? Justify your response.

Solution 5

(a) Mark scheme

- The scatterplot reveals a strong, positive, roughly linear association between chest circumference and weight of male tule elk. There are no points that seriously deviate from the straight-line pattern of the points in the plot. Full credit needs a description that includes at least 3 of components 1-4 AND component 5: (1) direction of association (positive/increasing), (2) strength of association (strong), (3) form of association (linear or approximately linear), (4) unusual features (no points with large discrepancies from the straight-line pattern exhibited by most points - it is sufficient to simply state there are no unusual features/outliers), (5) context - association between chest circumference and weight of tule elk (minimally sufficient to refer to the association between chest

Solution 5

circumference/measurement and weight without explicitly naming tule elk).
Partial credit if the response satisfies only 1-2 of components 1-4 AND
component 5, OR at least 2 of components 1-4 but not component 5.

Solution 5

(b)(i) Mark scheme

- Using the least-squares regression equation $\text{predicted weight} = -350.3 + 3.7455(\text{chest circumference})$, the predicted weight of a male tule elk with a chest circumference of 145.9 cm is $-350.3 + 3.7455(145.9) \approx 196.17$ kg. NOTE: parts (b)(i) and (b)(ii) are scored TOGETHER as ONE holistic unit in the official scoring guidelines, so this leaf carries this part's full mark. Full credit for the combined part (b) needs both: (1) provides the correct predicted value 196.17 kg in part (b)(i) with work shown, (2) provides the correct residual 8.13 kg in part (b)(ii) [graded together with leaf 5(b)(ii)], or a value consistent with the predicted weight calculated here, with work shown. Partial credit if the response satisfies only one of the two components, OR gives both correct values with no work

Solution 5

shown. A response using an undefined or incorrect variable to identify the work shown (e.g. x or p -hat instead of predicted weight) does not satisfy component 1. An arithmetic or transcription error can be ignored if correct work is shown.

Solution 5

(b)(ii) Mark scheme

- The residual for the male tule elk with chest circumference 145.9 cm and actual weight 204.3 kg is $\text{residual} = \text{actual} - \text{predicted} = 204.3 - 196.17 \approx 8.13$ kg. This leaf's credit is bundled with 5(b)(i) in the official scoring guidelines (Part (b) of the SG scores both sub-parts together as one holistic unit), so it carries 0 of this question's marks on its own - but a correct residual of 8.13 kg (or a value consistent with the predicted weight from part (b)(i)), with work shown, is still expected to complete credit for part (b). An arithmetic or transcription error can be ignored if correct work is shown.

Solution 5

(c) Mark scheme

- The value of the slope of the least-squares regression line is 3.7455 (may be rounded to 3.746, 3.75, or 3.7, but not to 3 or 4). This value indicates that the predicted weight of a tule elk increases by 3.7455 kilograms for each additional centimeter of chest circumference. Full credit needs all 3 components: (1) identifies the value of the slope as 3.7455 (a response that only contains 3.7455 in the interpretation still satisfies this), (2) provides an interpretation that references an increase of a number of kilograms of weight for each one-centimeter increase in chest circumference (units must be correctly specified for both weight and length), (3) indicates the slope represents a change in a PREDICTION using non-deterministic language such as 'predicted,' 'estimated,' 'expected,' or

Solution 5

'average' (not a deterministic claim that weight definitely increases). Partial credit if only 2 of the 3 components are satisfied. It is not required to refer specifically to the 'least-squares regression line.'

Solution 5

(d)(i) Mark scheme

- The degrees of freedom for the test of the population slope (relating chest circumference and weight for male tule elk, testing $H_0: \beta=4.5$ against a two-sided alternative, versus the sambar species' known slope of 4.5 kg/cm) are $n - 2 = 30 - 2 = 28$. The t-table shows that for 28 degrees of freedom, the p-value for a one-sided test would be 0.001. Because this is a two-sided test, the p-value is $(2)(0.001) = 0.002$. NOTE: parts (d)(i) and (d)(ii) are scored TOGETHER as ONE holistic unit in the official scoring guidelines, so this leaf carries this part's full mark. Full credit for the combined part (d) needs all 3: (1) gives a correct p-value of 0.002, consistent with the two-sided alternative hypothesis, (2) provides a correct comparison of the p-value to alpha AND a correct decision about the

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null/alternative hypothesis [graded together with leaf 5(d)(ii)], (3) provides a conclusion statement in context, consistent with and in terms of the alternative hypothesis, using non-deterministic language [also from part (d)(ii)]. Partial credit if only 2 of the 3 components are satisfied.

Solution 5

(d)(ii) Mark scheme

- Because the p-value (0.002) is less than $\alpha = 0.05$, reject the null hypothesis. There is sufficient statistical evidence that the population slope for the linear regression of weight vs. chest circumference for male tule elk is different from 4.5 kg/cm (i.e. different from the population slope for male sambars). This leaf's credit is bundled with 5(d)(i) in the official scoring guidelines (Part (d) of the SG scores both sub-parts together as one holistic unit), so it carries 0 of this question's marks on its own - but a correct decision (reject H_0) with a correctly-contextualized, non-deterministic conclusion is still expected to complete credit for part (d). An explicit decision statement is not strictly required if the decision is clearly implied within the conclusion.

Question 1

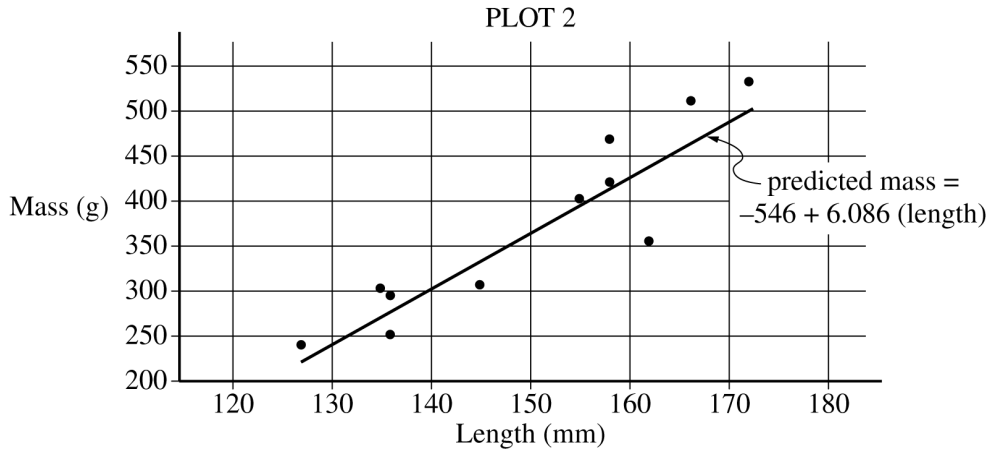
2022 ■ Q1

A biologist gathered data on the length, in millimeters (mm), and the mass, in grams (g), for 11 bullfrogs. The data are shown in Plot 1.

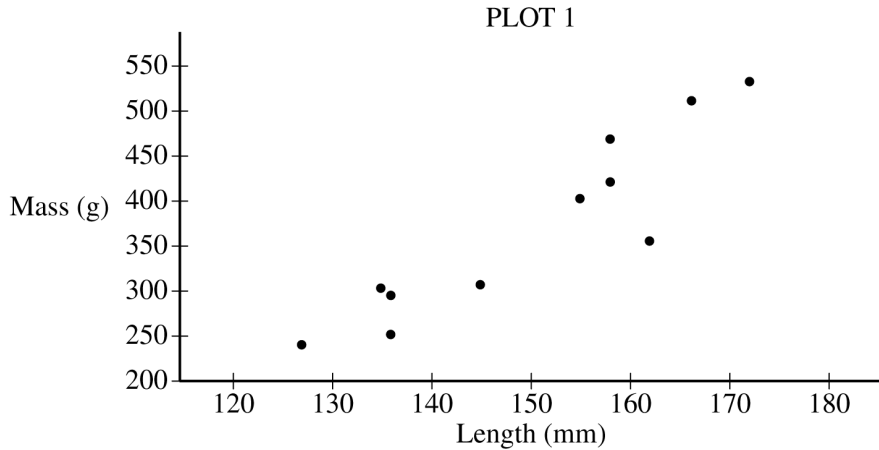
[Plot 1: Scatterplot titled “PLOT 1”; x-axis “Length (mm)” from 120 to 180; y-axis “Mass (g)” from 200 to 550. Approximate points (length, mass): (127, 240), (135, 295), (136, 300), (136, 255), (145, 310), (155, 400), (157, 470), (158, 420), (162, 355), (166, 510), (172, 530).]

(a) Based on the scatterplot, describe the relationship between mass and length, in context.

Question 1



Question 1



Question 1

2022 ■ Q1

A biologist gathered data on the length, in millimeters (mm), and the mass, in grams (g), for 11 bullfrogs. The data are shown in Plot 1.

[Plot 1: Scatterplot titled “PLOT 1”; x-axis “Length (mm)” from 120 to 180; y-axis “Mass (g)” from 200 to 550. Approximate points (length, mass): (127, 240), (135, 295), (136, 300), (136, 255), (145, 310), (155, 400), (157, 470), (158, 420), (162, 355), (166, 510), (172, 530).]

(b) From the data, the biologist calculated the least-squares regression line for predicting mass from length. The least-squares regression line is shown in Plot 2.

[Plot 2: Same scatterplot as Plot 1 with a least-squares regression line drawn through the points, labeled “predicted mass = $-546 + 6.086(\text{length})$ ”; x-axis “Length (mm)” from 120 to 180, y-axis “Mass (g)” from 200 to 550.]

Identify and interpret the slope of the least-squares regression line in context.

Question 1

2022 ■ Q1

A biologist gathered data on the length, in millimeters (mm), and the mass, in grams (g), for 11 bullfrogs. The data are shown in Plot 1.

[Plot 1: Scatterplot titled “PLOT 1”; x-axis “Length (mm)” from 120 to 180; y-axis “Mass (g)” from 200 to 550. Approximate points (length, mass): (127, 240), (135, 295), (136, 300), (136, 255), (145, 310), (155, 400), (157, 470), (158, 420), (162, 355), (166, 510), (172, 530).]

(c) Interpret the coefficient of determination of the least-squares regression line, $r^2 \approx 0.819$, in context.

Question 1

2022 ■ Q1

A biologist gathered data on the length, in millimeters (mm), and the mass, in grams (g), for 11 bullfrogs. The data are shown in Plot 1.

[Plot 1: Scatterplot titled “PLOT 1”; x-axis “Length (mm)” from 120 to 180; y-axis “Mass (g)” from 200 to 550. Approximate points (length, mass): (127, 240), (135, 295), (136, 300), (136, 255), (145, 310), (155, 400), (157, 470), (158, 420), (162, 355), (166, 510), (172, 530).]

(d)(i) From Plot 2, consider the residuals of the 11 bullfrogs.

Based on the plot, approximately what is the length and mass of the bullfrog with the largest absolute value residual?

(d)(ii) Does the least-squares regression line overestimate or underestimate the mass of the bullfrog identified in part (d-i)? Explain your answer.

Solution 1

(a) Mark scheme

- The scatterplot reveals a strong, positive, roughly linear association between the mass and length of bullfrogs. There are no points that seriously deviate from the straight-line pattern of the points in the plot. Full credit needs a description including at least three of: (1) direction of association (positive/increasing), (2) strength of association (strong), (3) form (linear or approximately linear), (4) no unusual features (no points with large discrepancies from the pattern), plus (5) context (association between length and mass of bullfrogs, or between length and mass in general).

Solution 1

(b) Mark scheme

- The slope of the least-squares regression line is 6.086 (may be rounded to 6.09 or 6.1, but not to 6). This indicates the predicted mass of a bullfrog increases by 6.086 grams for each additional millimeter of length. Full credit needs all three: (1) identifies the slope value as 6.086, (2) an interpretation referencing an increase of a number of grams of mass for each one-millimeter increase in length (correct units for both mass and length), (3) uses non-deterministic language such as 'predicted', 'estimated', 'expected', or 'average' rather than stating the change as certain.

Solution 1

(c) Mark scheme

- The coefficient of determination is $r^2 \approx 0.819$. This value indicates that 81.9% of the variation in bullfrog mass can be explained by variation in bullfrog length as described by the least-squares regression line. The interpretation must explicitly relate to the dependent (response) variable, mass, being explained by the independent (explanatory) variable, length – a generic definition with no context, an interpretation of r (not r^2) instead, or an interpretation that references the predicted y -values or the data (rather than mass explained by length) earns only partial credit.

Solution 1

(d)(i)

Mark scheme

- From Plot 2, the largest residual in absolute value belongs to the bullfrog with length approximately 162 millimeters and mass approximately 356 grams (any length between 160 and 165 mm and mass between 350 and 375 g, read from the plot, is accepted as reasonable). This identification is the first of two components that together earn the single point shared with part (d-ii): full credit for that shared point additionally requires correctly stating in (d-ii) whether the regression line overestimates or underestimates this bullfrog's mass.

Solution 1

(d)(ii) Mark scheme

- The least-squares regression line overestimates the mass of the bullfrog identified in part (d-i): Plot 2 shows the point for the bullfrog with length 162 mm lies below the least-squares regression line (the predicted mass at length 162, using $-546 + (6.086)(162) = 439.9$ grams, is larger than the bullfrog's actual mass of about 356 grams). This single point is shared between (d-i) and (d-ii) and requires both: (1) correctly identifying the bullfrog in (d-i) (length between 160-165 mm, mass between 350-375 g), and (2) explicitly stating in (d-ii) whether the line overestimates or underestimates the mass, with a justification comparing the identified point to the regression line (a numerical predicted-mass value is not required, but if given must be reasonable, e.g. between 425 and 450 g).

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2021 ■ Q6

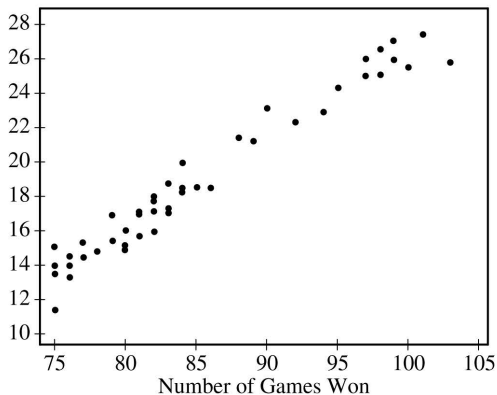
Attendance at games for a certain baseball team is being investigated by the team owner. The following boxplots summarize the attendance, measured as average number of attendees per game, for 47 years of the team's existence. The boxplots include the 30 years of games played in the old stadium and the 17 years played in the new stadium.

[Two vertical boxplots side by side, y-axis "Average Attendance (thousands)" from 10 to 28 in increments of 2. "Old Stadium" boxplot: whisker from about 11.5 to 20, box (Q1 to Q3) from about 14 to 17.5, median line at about 16. "New Stadium" boxplot: whisker from about 16 to 27.5, box (Q1 to Q3) from about 22 to 26.5, median line at about 25.]

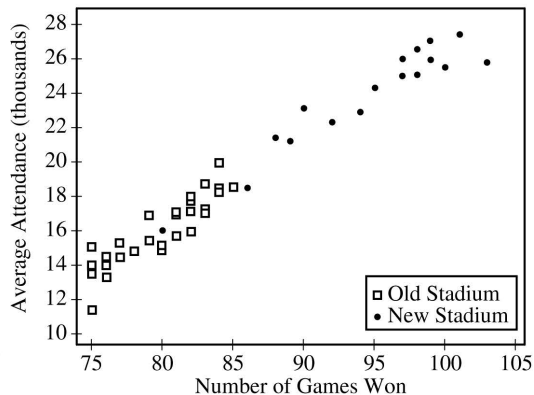
(a) Compare the distributions of average attendance between the old and new stadiums.

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Graph I



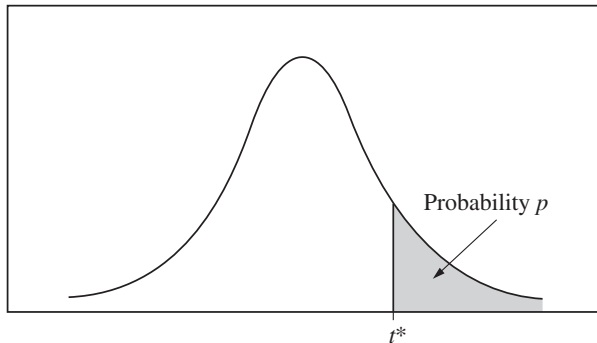
Graph II



Question 6

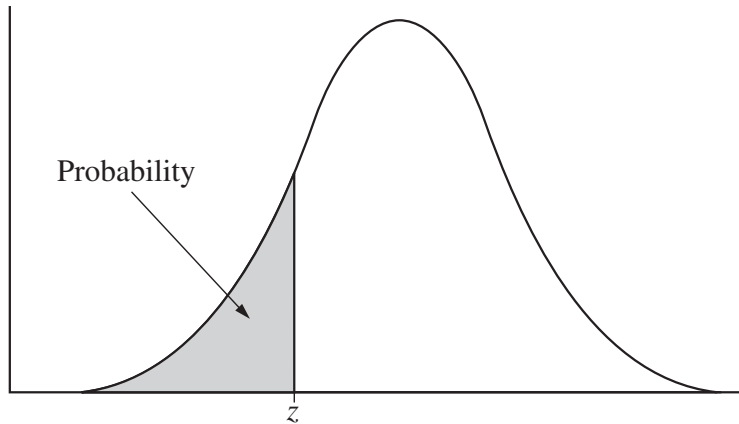
3.0	.9981	.9981	.9981	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^* .



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Table entry for z is the probability lying below z .



Question 6

2021 ■ Q6

Attendance at games for a certain baseball team is being investigated by the team owner. The following boxplots summarize the attendance, measured as average number of attendees per game, for 47 years of the team's existence. The boxplots include the 30 years of games played in the old stadium and the 17 years played in the new stadium.

[Two vertical boxplots side by side, y-axis "Average Attendance (thousands)" from 10 to 28 in increments of 2. "Old Stadium" boxplot: whisker from about 11.5 to 20, box (Q1 to Q3) from about 14 to 17.5, median line at about 16. "New Stadium" boxplot: whisker from about 16 to 27.5, box (Q1 to Q3) from about 22 to 26.5, median line at about 25.]

(b) The following scatterplot shows average attendance versus year.

[Scatterplot titled with y-axis "Average Attendance (thousands)" from 10 to 28 in increments of 2, x-axis "Year" from 1970 to 2020 in increments of 10 (gridlines at 1970, 1980, 1990, 2000, 2010, 2020). Legend distinguishes open squares = Old

Question 6

Stadium and filled circles = New Stadium. Old Stadium square points span roughly 1970-2000, scattered between about 11 and 20 with no strong trend (values fluctuate between roughly 13-20 throughout). New Stadium circle points span roughly 2000-2018, showing a clear upward trend from about 16-21 in the early 2000s rising steadily to about 26-27.5 by the late 2010s.]

Compare the trends in average attendance over time between the old and new stadium.

Question 6

2021 ■ Q6

Attendance at games for a certain baseball team is being investigated by the team owner. The following boxplots summarize the attendance, measured as average number of attendees per game, for 47 years of the team's existence. The boxplots include the 30 years of games played in the old stadium and the 17 years played in the new stadium.

[Two vertical boxplots side by side, y-axis "Average Attendance (thousands)" from 10 to 28 in increments of 2. "Old Stadium" boxplot: whisker from about 11.5 to 20, box (Q1 to Q3) from about 14 to 17.5, median line at about 16. "New Stadium" boxplot: whisker from about 16 to 27.5, box (Q1 to Q3) from about 22 to 26.5, median line at about 25.]

(c) Consider the following scatterplots.

[Graph I: scatterplot titled "Graph I", y-axis "Average Attendance (thousands)" (label shown on Graph II) from 10 to 28 in increments of 2, x-axis "Number of Games Won" from 75 to 105 in increments of 5. All points plotted as plain dots (stadium not

Question 6

distinguished) showing a positive, fairly linear association: points cluster from about (75,11-15) at the low end rising to about (100-103,26-28) at the high end.]

[Graph II: scatterplot titled “Graph II”, same axes as Graph I (“Average Attendance (thousands)” from 10 to 28, “Number of Games Won” from 75 to 105). Same data as Graph I but now marked with open squares = Old Stadium and filled circles = New Stadium (legend shown). Old Stadium (square) points cluster in the lower-left region, roughly games won 75-86 and attendance 11-20. New Stadium (circle) points cluster in the upper-right region, roughly games won 75-103 and attendance 16-27.5, appearing to lie along a steeper positive trend than the Old Stadium points.]

(c)(i) Graph I shows the average attendance versus number of games won for each year. Describe the relationship between the variables.

(c)(ii) Graph II shows the same information as Graph I, but also indicates the old and new stadiums. Does Graph II suggest that the rate at which attendance changes as

Question 6

number of games won increases is different in the new stadium compared to the old stadium? Explain your reasoning.

Question 6

2021 ■ Q6

Attendance at games for a certain baseball team is being investigated by the team owner. The following boxplots summarize the attendance, measured as average number of attendees per game, for 47 years of the team's existence. The boxplots include the 30 years of games played in the old stadium and the 17 years played in the new stadium.

[Two vertical boxplots side by side, y-axis "Average Attendance (thousands)" from 10 to 28 in increments of 2. "Old Stadium" boxplot: whisker from about 11.5 to 20, box (Q1 to Q3) from about 14 to 17.5, median line at about 16. "New Stadium" boxplot: whisker from about 16 to 27.5, box (Q1 to Q3) from about 22 to 26.5, median line at about 25.]

(d) Consider the three variables: number of games won, year, and stadium. Based on the graphs, explain how one of those variables could be a confounding variable in the relationship between average attendance and the other variables.

Solution 6

(a) Mark scheme

- The boxplots show the new stadium tended to have higher average per-game attendance (median $\approx 25,000$) *than the old stadium* (median $\approx 16,000$). *The IQRs (spread/variability) are similar between the two stadiums, though the overall range of game attendance. Credit requires (1) a correct center comparison (new > old, with stadiums identified and an explicit comparison phrase), (2) a correct variability comparison (s*

Solution 6

(b) Mark scheme

- In the new stadium (years 2000-2016), average per-game attendance shows an increasing (positive) linear trend, rising from about 16,000 in 2000 to about 27,000 in 2016. In the old stadium (years 1970-1999), there is no clear increasing or decreasing trend – attendance fluctuates around an average of about 16,000 attendees per game with no obvious pattern over time. Credit requires (1) describing the new-stadium trend as increasing/positive, (2) describing the old-stadium trend as flat/no association (or positive but much less steep than the new stadium), AND (3) context identifying both stadiums, the explanatory variable (time/year), and the response variable (average attendance) or its units.

Solution 6

(c)

Mark scheme

- This leaf is the shared stem introducing Graph I (a plain scatterplot of average attendance vs. number of games won, stadium not distinguished) and Graph II (the same data, but marked by stadium: old vs. new) that parts (c)(i) and (c)(ii) below are based on. It poses no separate question of its own, so there is no independent scorable answer here – the graded reasoning about these two graphs is in leaves 6ci and 6cii.

Solution 6

(c)(i) Mark scheme

- Graph I shows a strong, positive, linear (or nearly linear) relationship between average per-game attendance and number of games won across the team's history: attendance increases roughly linearly with wins, by about 500 additional attendees per game for each additional game won, and the scatter about this linear trend is relatively small and fairly consistent (homogeneous) across the range of wins. Credit (part of a combined 5-component part-(c) score with 6cii) for describing direction (positive), form (linear), and strength (strong/moderately strong) of the relationship, ideally also noting the roughly constant variation about the trend.

Solution 6

(c)(ii) Mark scheme

- No – Graph II suggests the rate at which average attendance increases with games won is about the same for the two stadiums (or slightly larger for the old stadium): a line drawn through the old-stadium points would have roughly the same slope as (or a slightly steeper slope than) a line drawn through the new-stadium points, so there is no evidence the new stadium has a meaningfully different attendance-vs-wins rate than the old stadium. Credit (part of the same combined part-(c) score as 6ci) for correctly comparing the two implied slopes as about equal (or old $\geq$ new) *with an explanation referencing fitting a line through each stadium's points.*

Solution 6

(d) Mark scheme

- Number of games won could be a confounding variable for the effect of the new stadium on attendance. There is an association between attendance and stadium (part (a): new stadium had higher average attendance) and an association between attendance and games won (part (c): strong positive relationship); additionally, the team tended to win more games while playing in the new stadium than in the old stadium, so games won is also associated with stadium. Because these three variables are entangled, it is impossible to tell whether the higher attendance in the new stadium was actually caused by the stadium itself or simply by the team winning more games during that period – games won confounds the relationship between stadium and attendance. Credit requires (1)-(2) stating an

Solution 6

association between attendance and each of two explanatory variables (stadium, year, or wins), (3) stating an association between those same two explanatory variables, AND (4) explaining the confounding logic (the identified variable could be the true cause, or it's impossible to tell which variable is responsible).

Question 6

2016 ■ Q6

A newspaper in Germany reported that the more semesters needed to complete an academic program at the university, the greater the starting salary in the first year of a job. The report was based on a study that used a random sample of 24 people who had recently completed an academic program and were selected to represent the population of that country, in thousands of euros, for the first year of a job. The data are shown in the scatterplot below.

[Scatterplot titled with y-axis "Starting Salary (1,000 euros)" ranging approximately 25 to 65, x-axis "Number of Semesters" ranging approximately 0 to 20. Points scattered showing a general upward trend from lower-left (fewer semesters, lower salary, around 30) to upper-right (more semesters, higher salary, up to about 65), with considerable scatter throughout.]

The table below shows computer output from a linear regression analysis on the data.

Question 6

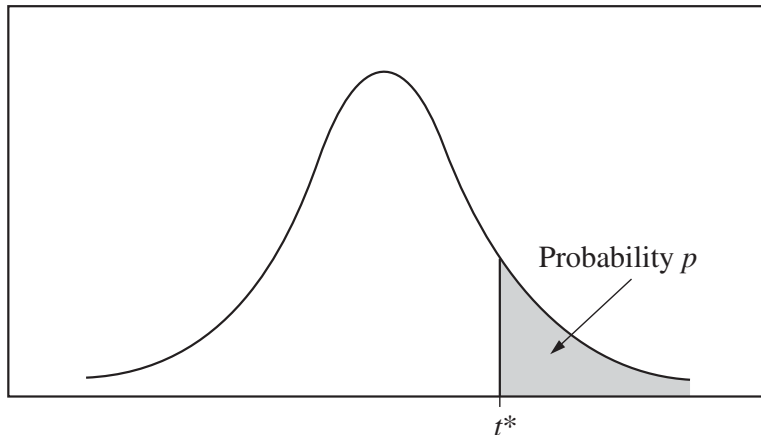
Predictor	Coef	SE Coef	T	P
Constant	34.018	4.455	7.64	0.000
Semesters	1.1594	0.3482	3.33	0.003

$S = 7.37702$ $R\text{-Sq} = 33.5\%$ $R\text{-Sq}(\text{adj}) = 30.5\%$

(a) Does the scatterplot support the newspaper's report about number of semesters and starting salary? Justify your answer.

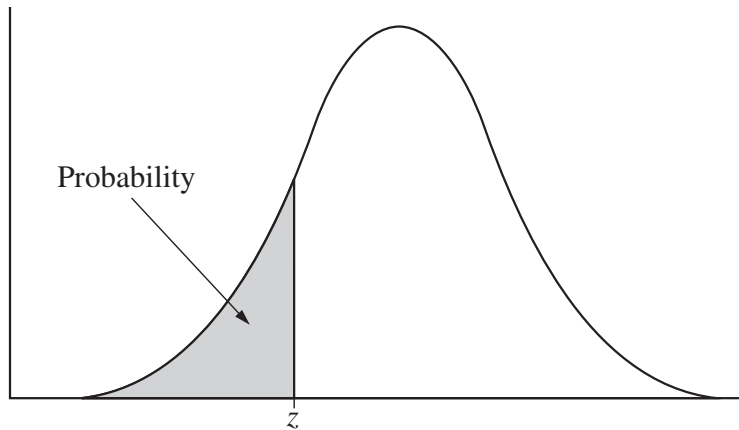
Question 6

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^* .

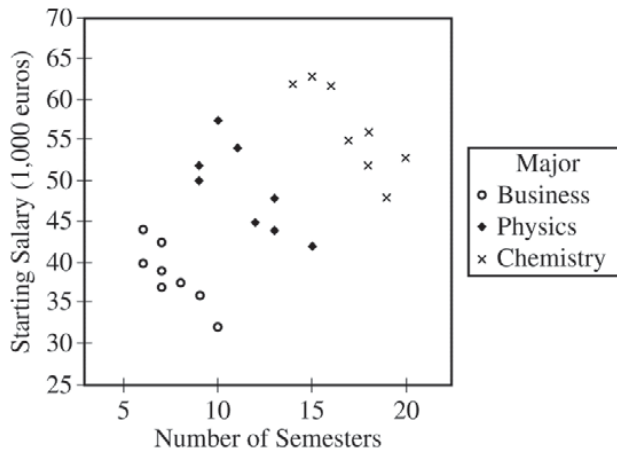


Question 6

Table entry for z is the probability lying below z .



Question 6



Question 6

2016 ■ Q6

A newspaper in Germany reported that the more semesters needed to complete an academic program at the university, the greater the starting salary in the first year of a job. The report was based on a study that used a random sample of 24 people who had recently completed an academic program and were selected to represent the population of that country, in thousands of euros, for the first year of a job. The data are shown in the scatterplot below.

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The table below shows computer output from a linear regression analysis on the data.

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Semesters	1.1594	0.3482	3.33	0.003						

$S = 7.37702$ $R\text{-Sq} = 33.5\%$ $R\text{-Sq}(\text{adj}) = 30.5\%$

Question 6

(b) Identify the slope of the least-squares regression line, and interpret the slope in context.

An independent researcher received the data from the newspaper and conducted a new analysis by separating the data into three groups based on the major of each person. A revised scatterplot identifying the major of each person is shown below.

[Revised scatterplot, same axes as before (y-axis “Starting Salary (1,000 euros)” approximately 25 to 65, x-axis “Number of Semesters” approximately 5 to 20), with points marked by three different symbols per the legend “Major: Business (square), Physics (circle), Chemistry (diamond)”. Business-major points cluster at lower semesters with lower-to-moderate salaries; Physics- and Chemistry-major points spread across higher semesters with a range of salaries, generally higher for more semesters.]

Question 6

2016 ■ Q6

A newspaper in Germany reported that the more semesters needed to complete an academic program at the university, the greater the starting salary in the first year of a job. The report was based on a study that used a random sample of 24 people who had recently completed an academic program and were selected to represent the population of that country, in thousands of euros, for the first year of a job. The data are shown in the scatterplot below.

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$S = 7.37702$ $R\text{-Sq} = 33.5\%$ $R\text{-Sq}(\text{adj}) = 30.5\%$

Question 6

(c) Based on the people in the sample, describe the association between starting salary and number of semesters for the business majors.

Question 6

2016 ■ Q6

A newspaper in Germany reported that the more semesters needed to complete an academic program at the university, the greater the starting salary in the first year of a job. The report was based on a study that used a random sample of 24 people who had recently completed an academic program and were selected to represent the population of that country, in thousands of euros, for the first year of a job. The data are shown in the scatterplot below.

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Question 6

(d) Based on the people in the sample, compare the median starting salaries for the three majors.

Question 6

2016 ■ Q6

A newspaper in Germany reported that the more semesters needed to complete an academic program at the university, the greater the starting salary in the first year of a job. The report was based on a study that used a random sample of 24 people who had recently completed an academic program and were selected to represent the population of that country, in thousands of euros, for the first year of a job. The data are shown in the scatterplot below.

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$S = 7.37702$ $R\text{-Sq} = 33.5\%$ $R\text{-Sq}(\text{adj}) = 30.5\%$

Question 6

(e) Based on the analysis conducted by the independent researcher, how should the newspaper report be modified to give a better description of the relationship between the number of semesters and the starting salary for the people in the sample?

Solution 6

(a)

Mark scheme

- Yes – the scatterplot supports the newspaper's report, because it shows a positive association between number of semesters needed to complete an academic program and starting salary (in general, needing more semesters is associated with a higher starting salary). Full credit requires both addressing the positive association AND using it to justify that the scatterplot supports the report.

Solution 6

(b)

Mark scheme

- The slope of the least-squares regression line is 1.1594 (from $\hat{y} = 34.018 + 1.1594x$). Interpretation: for each additional semester needed to complete an academic program, the predicted (model-estimated / average) starting salary increases by about €1,159.40 (equivalently 1.1594 thousand euros). Full credit requires the correct numerical slope value AND an in-context interpretation using nondeterministic language (e.g. "predicted"/"estimated"/"on average" – not a deterministic claim that salary WILL increase).

Solution 6

(c)

Mark scheme

- For the business majors alone, there is a strong, negative, linear association between number of semesters and starting salary: business majors who need a greater number of semesters to complete their academic program tend to have LOWER starting salaries. Full credit requires stating the association is negative, describing it as strong and/or linear, and referring to both variables (semesters, salary) in context.

Solution 6

(d)

Mark scheme

- Comparing the three majors by median starting salary: business majors have the lowest median starting salary (around €38,000), followed by physics majors (around €48,000), with chemistry majors having the highest median starting salary (around €55,000). Full credit requires a correct comparison of all three majors, with reasonable median-salary values (or explicit reference to "median starting salary").

Solution 6

(e) Mark scheme

- The newspaper's report should be modified to account for major: overall, majors that take longer to complete DO tend to have higher starting salaries (chemistry highest, physics next, business lowest – the original positive association holds across majors). However, WITHIN each individual major, students who take a greater number of semesters tend to have LOWER starting salaries (a negative association) – the opposite of the overall pattern. Full credit requires noting the negative within-major association for each major AND still acknowledging the overall positive association across majors (a Simpson's-paradox-type reversal).

Question 5

2015 ■ Q5

A student measured the heights and the arm spans, rounded to the nearest inch, of each person in a random sample of 12 seniors at a high school. A scatterplot of arm span versus height for the 12 seniors is shown.

[Scatterplot of Arm Span (inches) on the y-axis, 60 to 72, versus Height (inches) on the x-axis, 60 to 72. Approximate points: (61,64), (62,61), (64,64), (64,62), (65,67), (65,64), (66,69), (69,65), (69,71), (69,69), (72,72), (60,62).]

Let x represent height, in inches, and let y represent arm span, in inches. Two scatterplots of the same data are shown below. Graph 1 shows the data with the least squares regression line $\hat{y} = 11.74 + 0.8247x$, and graph 2 shows the data with the line $y = x$.

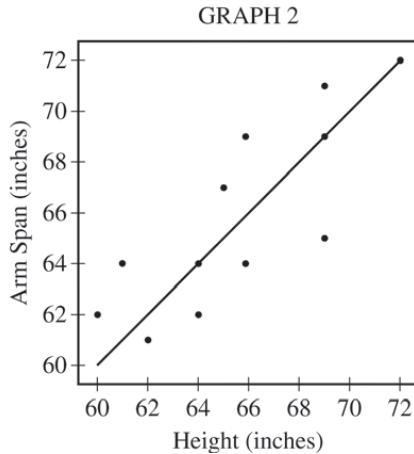
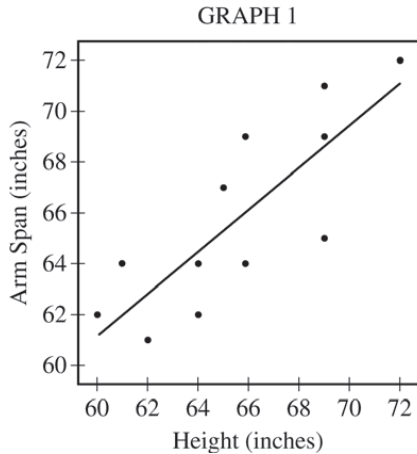
[Two side-by-side scatterplots, both with the same 12 points as above, axes Arm Span (inches) 60 to 72 vs. Height (inches) 60 to 72. GRAPH 1: shows the least-squares

Question 5

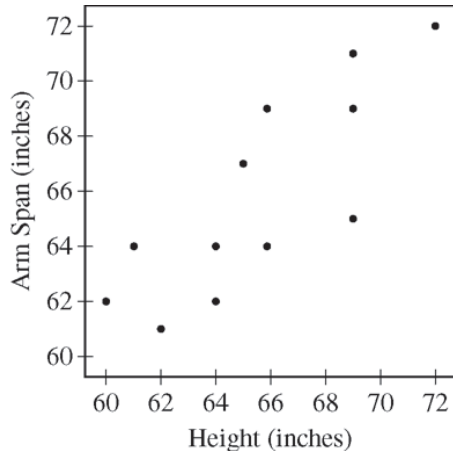
regression line $\hat{y} = 11.74 + 0.8247x$ passing through the data, from about (60,61) to (72,71). GRAPH 2: shows the line $y = x$ passing through the data, from (60,60) to (72,72).]

(a) Based on the scatterplot, describe the relationship between arm span and height for the sample of 12 seniors.

Question 5



Question 5



Question 5

Classification	Square	Tall Rectangle	Short Rectangle
Frequency			