Past-paper walk-through

Representing Two Categorical Variables ▪ 2.2

eXercise

Questions in this set

- 2024 Q2
- 2021 Q5
- 2014 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

2024 ▪ Q2

A local elementary school decided to sell bottles printed with the school district's logo as a fund-raiser. The students in the elementary school were asked to sell bottles in three different sizes (small, medium, and large). The relative frequencies of the number of bottles sold for each size by the elementary school were 0.5 for small bottles, 0.3 for medium bottles, and 0.2 for large bottles.

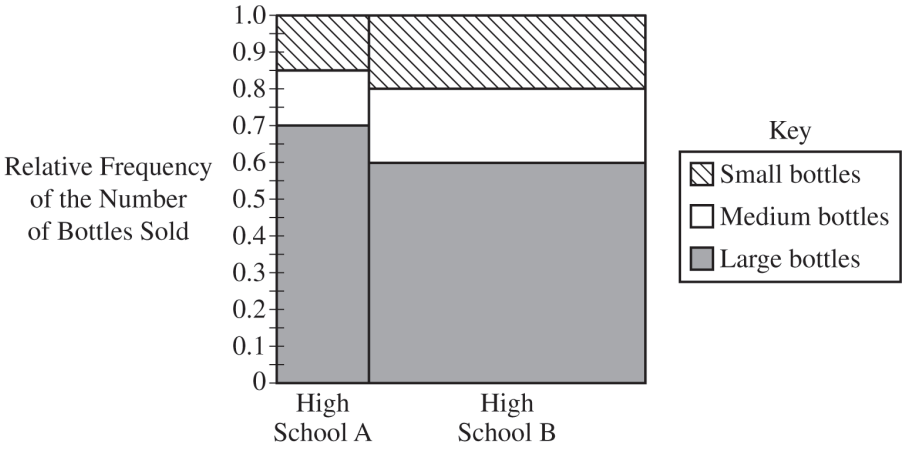
A local middle school also decided to sell bottles as a fund-raiser, using the same three sizes (small, medium, and large). The middle school students sold three times the number of bottles that the elementary school students sold. For the middle school students, the proportion of bottles sold was equal for all three sizes.

(a) Complete the segmented bar graphs representing the relative frequencies of the number of bottles sold for each size by students at each school.

Question 2

[Two blank segmented (stacked) bar graphs to be completed by the student, each a single horizontal bar on an axis "Relative Frequency of Number of Bottles Sold" from 0 to 1.0 (gridlines at 0.2, 0.4, 0.6, 0.8, 1.0): one labeled "Elementary School", one labeled "Middle School". A key at the right shows three segment patterns: hatched/cross-hatched = Small bottles, blank/white = Medium bottles, gray/shaded = Large bottles.]

Question 2


Relative Frequency of the Number of Bottles Sold
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
High School A
High School B
Key
Small bottles
Medium bottles
Large bottles

Question 2

2024 ▪ Q2

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A local middle school also decided to sell bottles as a fund-raiser, using the same three sizes (small, medium, and large). The middle school students sold three times the number of bottles that the elementary school students sold. For the middle school students, the proportion of bottles sold was equal for all three sizes.

(b) An administrator at the elementary school concluded that the elementary school students sold more small bottles than the middle school students did. Is the elementary school administrator's conclusion correct? Explain your response.

Question 2

2024 ▪ Q2

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A local middle school also decided to sell bottles as a fund-raiser, using the same three sizes (small, medium, and large). The middle school students sold three times the number of bottles that the elementary school students sold. For the middle school students, the proportion of bottles sold was equal for all three sizes.

(c)(i) Two high schools are also selling the bottles and are competing to see which one sold more large bottles.

Question 2

A mosaic plot for the distribution of the number of bottles sold by each of the high schools is shown here.

[Mosaic plot titled "Distribution of the Number of Bottles Sold by High School"; y-axis "Relative Frequency of the Number of Bottles Sold" from 0 to 1.0 in increments of 0.1; two columns, "High School A" and "High School B", with widths proportional to the total number of bottles sold (High School A narrower, High School B wider). Key: hatched = Small bottles, blank/white = Medium bottles, gray/shaded = Large bottles. For High School A (reading bottom to top): Large bottles (gray) from 0 to about 0.7; Medium bottles (blank) from about 0.7 to 0.85; Small bottles (hatched) from about 0.85 to 1.0. For High School B (reading bottom to top): Large bottles (gray) from 0 to 0.6; Medium bottles (blank) from 0.6 to 0.8; Small bottles (hatched) from 0.8 to 1.0.]

Question 2

Which of the two high schools sold a greater proportion of large bottles? Justify your answer.

(c)(ii) Which of the two high schools sold a greater number of large bottles? Justify your answer.

Solution 2

(a) Mark scheme

- Complete each bar so the segments' widths equal the actual relative frequencies for that school: the Elementary School bar is partitioned into small/medium/large sections matching the elementary data (using the given hatched = small, blank = medium, gray = large key), and the Middle School bar is partitioned into three EQUAL-width sections (each representing a relative frequency of about $1/3 \approx 0.33$) for small/medium/large, since the middle school sold the same number of each size. Full credit requires both bars correctly partitioned – the elementary bar per its true proportions AND the middle-school bar into three equal areas. The order of segments (e.g. small-medium-large vs. large-medium-small) does not

Solution 2

matter, a response may supply its own key defining small/medium/large instead of the given one, and cutoffs within ± 0.05 of the actual value are acceptable.

Solution 2

(b) Mark scheme

- No – the elementary administrator's conclusion is not correct. The segment for small bottles is wider (a greater relative frequency) for the elementary school than for the middle school, but the middle school sold three times as many total bottles as the elementary school. If the elementary school sold x bottles total, it sold $0.5x$ small bottles; the middle school sold $3x$ bottles total, of which $0.\overline{3}(3x) = x$ were small. Example: if the elementary school sold 100 bottles total, it sold $0.5(100) = 50$ small bottles, while the middle school sold 300 bottles total, of which $(0.\overline{3})(300) = 100$ were small; since $100 > 50$, the middle school actually sold MORE small bottles in absolute count even though its proportion of small bottles is lower. Full credit needs: (1) states the elementary administrator is

Solution 2

incorrect, (2) correct mathematical support verifying the middle school sold more small bottles (consistent with part (a)), (3) context – the words "elementary," "middle," and "bottles" must appear.

Solution 2

(c)(i)

Mark scheme

- High School A sold the greater PROPORTION of large bottles. From the mosaic plot, the relative frequency (height) of the large-bottles (gray) segment is 0.7 for High School A and 0.6 for High School B, and $0.7 > 0.6$. Full credit needs the correct school identified (A) AND the justification based on the height/relative frequency of the large-bottle segments (if the response refers to "proportions" rather than relative frequencies, it must give the values 0.7 and 0.6).

Solution 2

(c)(ii) Mark scheme

- High School B sold the greater NUMBER of large bottles. The number of bottles sold is represented by the AREA of the shaded region, not its height/proportion – High School B's column is wider (more total bottles sold) than High School A's, so even though a smaller share (0.6 vs 0.7) of High School B's bottles were large, the area of the rectangle representing large bottles sold by High School B is clearly larger than the area for High School A. Full credit needs High School B identified AND the justification based on comparing the AREAS of the large-bottle rectangles (reasonable estimates of the areas are acceptable).

Question 5

2021 ▪ Q5

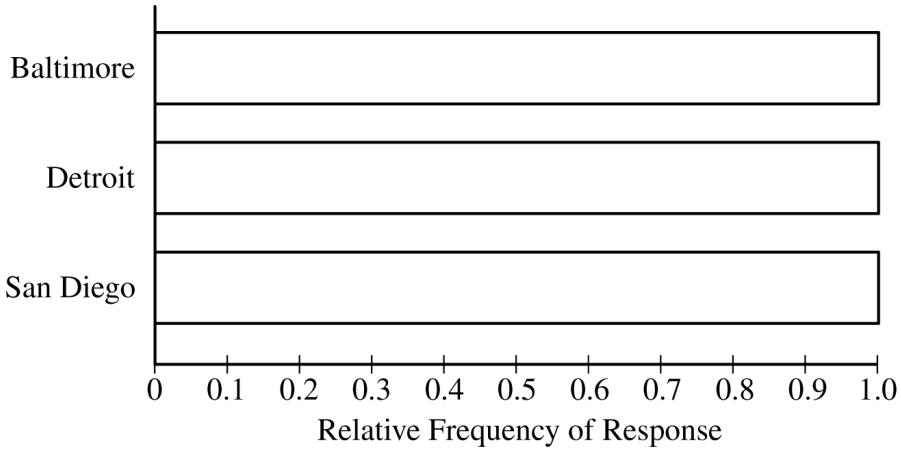
A research center conducted a national survey about teenage behavior. Teens were asked whether they had consumed a soft drink in the past week. The following table shows the counts for three independent random samples from major cities.

	Baltimore	**Detroit**	**San Diego**	**Total**
Yes	727	1,232	1,482	3,441
No	177	431	798	1,406
Total	904	1,663	2,280	4,847

Question 5

(a) Suppose one teen is randomly selected from each city's sample. A researcher claims that the likelihood of selecting a teen from Baltimore who consumed a soft drink in the past week is less than the likelihood of selecting a teen from either one of the other cities who consumed a soft drink in the past week because Baltimore has the least number of teens who consumed a soft drink. Is the researcher's claim correct? Explain your answer.

Question 5


Baltimore
Detroit
San Diego
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Relative Frequency of Response