

Past-paper walk-through

Introducing Statistics: Are Variables Related? ■ 2.1

eXercise

Questions in this set

- 2022 Q1
- 2017 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- **//** separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 1

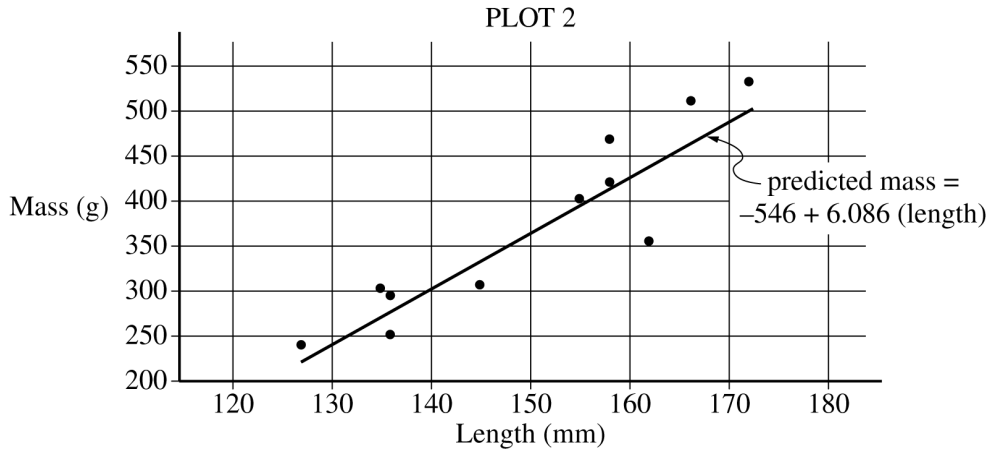
2022 ■ Q1

A biologist gathered data on the length, in millimeters (mm), and the mass, in grams (g), for 11 bullfrogs. The data are shown in Plot 1.

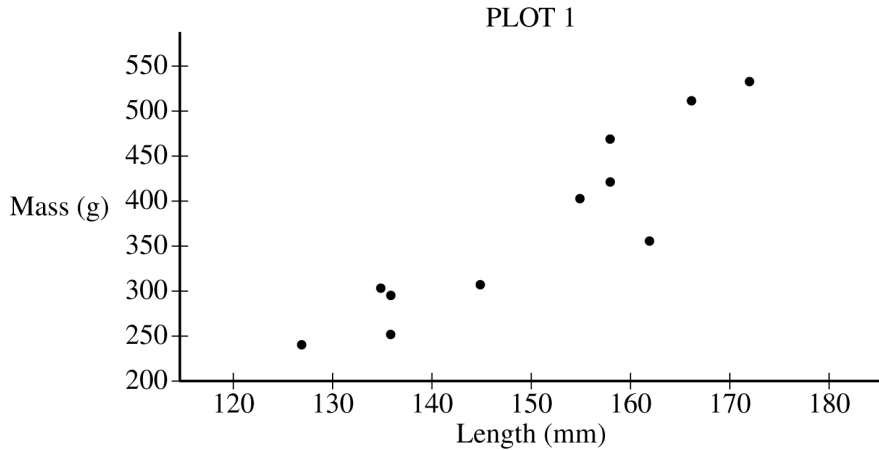
[Plot 1: Scatterplot titled “PLOT 1”; x-axis “Length (mm)” from 120 to 180; y-axis “Mass (g)” from 200 to 550. Approximate points (length, mass): (127, 240), (135, 295), (136, 300), (136, 255), (145, 310), (155, 400), (157, 470), (158, 420), (162, 355), (166, 510), (172, 530).]

(a) Based on the scatterplot, describe the relationship between mass and length, in context.

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(b) From the data, the biologist calculated the least-squares regression line for predicting mass from length. The least-squares regression line is shown in Plot 2.

[Plot 2: Same scatterplot as Plot 1 with a least-squares regression line drawn through the points, labeled “predicted mass = $-546 + 6.086(\text{length})$ ”; x-axis “Length (mm)” from 120 to 180, y-axis “Mass (g)” from 200 to 550.]

Identify and interpret the slope of the least-squares regression line in context.

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(c) Interpret the coefficient of determination of the least-squares regression line, $r^2 \approx 0.819$, in context.

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(d)(i) From Plot 2, consider the residuals of the 11 bullfrogs.

Based on the plot, approximately what is the length and mass of the bullfrog with the largest absolute value residual?

(d)(ii) Does the least-squares regression line overestimate or underestimate the mass of the bullfrog identified in part (d-i)? Explain your answer.

Solution 1

(a) Mark scheme

- The scatterplot reveals a strong, positive, roughly linear association between the mass and length of bullfrogs. There are no points that seriously deviate from the straight-line pattern of the points in the plot. Full credit needs a description including at least three of: (1) direction of association (positive/increasing), (2) strength of association (strong), (3) form (linear or approximately linear), (4) no unusual features (no points with large discrepancies from the pattern), plus (5) context (association between length and mass of bullfrogs, or between length and mass in general).

Solution 1

(b) Mark scheme

- The slope of the least-squares regression line is 6.086 (may be rounded to 6.09 or 6.1, but not to 6). This indicates the predicted mass of a bullfrog increases by 6.086 grams for each additional millimeter of length. Full credit needs all three: (1) identifies the slope value as 6.086, (2) an interpretation referencing an increase of a number of grams of mass for each one-millimeter increase in length (correct units for both mass and length), (3) uses non-deterministic language such as 'predicted', 'estimated', 'expected', or 'average' rather than stating the change as certain.

Solution 1

(c) Mark scheme

- The coefficient of determination is $r^2 \approx 0.819$. This value indicates that 81.9% of the variation in bullfrog mass can be explained by variation in bullfrog length as described by the least-squares regression line. The interpretation must explicitly relate to the dependent (response) variable, mass, being explained by the independent (explanatory) variable, length – a generic definition with no context, an interpretation of r (not r^2) instead, or an interpretation that references the predicted y -values or the data (rather than mass explained by length) earns only partial credit.

Solution 1

(d)(i)

Mark scheme

- From Plot 2, the largest residual in absolute value belongs to the bullfrog with length approximately 162 millimeters and mass approximately 356 grams (any length between 160 and 165 mm and mass between 350 and 375 g, read from the plot, is accepted as reasonable). This identification is the first of two components that together earn the single point shared with part (d-ii): full credit for that shared point additionally requires correctly stating in (d-ii) whether the regression line overestimates or underestimates this bullfrog's mass.

Solution 1

(d)(ii) Mark scheme

- The least-squares regression line overestimates the mass of the bullfrog identified in part (d-i): Plot 2 shows the point for the bullfrog with length 162 mm lies below the least-squares regression line (the predicted mass at length 162, using $-546 + (6.086)(162) = 439.9$ grams, is larger than the bullfrog's actual mass of about 356 grams). This single point is shared between (d-i) and (d-ii) and requires both: (1) correctly identifying the bullfrog in (d-i) (length between 160-165 mm, mass between 350-375 g), and (2) explicitly stating in (d-ii) whether the line overestimates or underestimates the mass, with a justification comparing the identified point to the regression line (a numerical predicted-mass value is not required, but if given must be reasonable, e.g. between 425 and 450 g).

Question 1

2017 ■ Q1

Researchers studying a pack of gray wolves in North America collected data on the length x , in meters, from nose to tip of tail, and the weight y , in kilograms, of the wolves. A scatterplot of weight versus length revealed a relationship between the two variables described as positive, linear, and strong.

(a)(i) For the situation described above, explain what is meant by each of the following words.

Positive:

(a)(ii) Linear:

(a)(iii) Strong:

(b) The data collected from the wolves were used to create the least-squares equation $\hat{y} = -16.46 + 35.02x$.

Interpret the meaning of the slope of the least-squares regression line in context.

Question 1

(c) One wolf in the pack with a length of 1.4 meters had a residual of -9.67 kilograms. What was the weight of the wolf?

Solution 1

(a)(i) Mark scheme

- 'Positive' means that wolves with higher values of length also tend to have higher values of weight – low length goes with low weight, and high length goes with high weight (as length increases, weight tends to increase). Acceptable phrasing: "As length increases, so does weight"; "Longer wolves weigh more"; "The points on the graph go up as you move left to right." NOT acceptable: "As length goes up, weight changes"; "the correlation is greater than 0" (must describe the low-low/high-high pattern, not just restate the term). Full part (a) credit needs a definition like this for positive, one for linear, one for strong, AND at least one of the three stated in context (using length/weight or meters/kilograms).

Solution 1

(a)(ii) Mark scheme

- 'Linear' means that as length increases by one meter, weight tends to change by a constant amount, on average – the data points follow the pattern of a straight line (equivalently, the rate of change of weight with respect to length is constant). Acceptable: "The points generally follow a straight line"; "the relationship between x and y is straight"; "length and weight have a constant slope." NOT acceptable: "the points all line up"; "you can draw a straight line through the points"; "there is a positive correlation"; merely restating that every increase in x yields a 35.02 increase in y . See the context/four-component requirement noted under 1ai for part (a) as a whole.

Solution 1

(a)(iii) Mark scheme

- 'Strong' means the data points fall close to the line (or curve) – observed weights are close to predicted weights, i.e. residuals (deviations) from the least-squares line are small, or equivalently the correlation coefficient is close to 1 in magnitude. Acceptable: "Observed values are close to predicted values"; "Deviations from the least-squares regression line are small"; "The correlation coefficient is close to 1." NOT acceptable: "the points are close together"; "the scatterplots are clustered together"; "there is a high positive correlation" (these describe density of points, not closeness to a line). See the context/four-component requirement noted under 1ai for part (a) as a whole.

Solution 1

(b) Mark scheme

- The slope of 35.02 means: two wolves whose lengths differ by one meter are predicted to differ in weight by 35.02 kilograms, with the longer wolf having the greater weight. Full credit requires three components: (1) correctly cites the slope value 35.02; (2) interprets it as a specified increase in weight for each unit increase in length; (3) indicates the relationship is not exact/deterministic, using language such as "predicted" or "on average" (e.g. "...predicted to differ by 35.02 kg" or "...weight increases by 35.02 kg on average"). A response that only says "weight increases by 35.02 kg per meter" with no averaging/predicted language misses component 3. Note: if the response instead identifies -16.46 (the

Solution 1

intercept) as the slope, component 2 is satisfied only if it states weight DECREASES by 16.46 kg on average for each meter increase.

Solution 1

(c) Mark scheme

- In general, $\text{residual} = \text{actual weight} - \text{predicted weight}$, so $\text{actual weight} = \text{predicted weight} + \text{residual}$. For the wolf with length 1.4 m: $\text{predicted weight} = -16.46 + 35.02(1.4) = 32.568$ kilograms. $\text{Actual weight} = 32.568 + (-9.67) = 22.898$ kilograms (≈ 22.9 kg). Full credit needs both: (1) the correct predicted-weight computation (32.568 kg), and (2) the correct actual-weight computation obtained by ADDING the (negative) residual to the predicted value, giving 22.9 kg. A common error is subtracting instead of adding the residual (giving 42.24 kg), which does not earn full credit. Minor arithmetic slips that don't lead to an unreasonable value (e.g. a negative weight) can still be treated as showing the correct method.