# Past-paper walk-through

## Summary Statistics for a Quantitative Variable ▪ 1.7

eXercise

# Questions in this set

- 2026 Q1
- 2025 Q1
- 2024 Q6
- 2021 Q1
- 2019 Q1
- 2019 Q6
- 2018 Q5
- 2016 Q1
- 2014 Q4

# Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

# Question 1

2026 ▪ Q1

A goat farmer raises two breeds of goats, Breed H and Breed J. He wants to compare the weights of the two breeds and takes independent random samples of 14 goats from each breed. The weight, in pounds, of each goat is measured. The weights of the 14 Breed H goats are recorded.

Weights of Breed H Goats (pounds)

48, 48, 55, 56, 56, 57, 62, 66, 72, 72, 72, 73, 80, 80

**(a)** Use the data in the list to determine the five-number summary for the sample of Breed H goats.

# Question 1

2026 ▪ Q1

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Weights of Breed H Goats (pounds)

48, 48, 55, 56, 56, 57, 62, 66, 72, 72, 72, 73, 80, 80

**(b)** The distribution of weight for Breed J goats is displayed in the boxplot.

[Boxplot titled "Weights of Breed J Goats (pounds)"; horizontal axis "Weight (pounds)" from 50 to 90, gridlines at 50, 55, 60, 65, 70, 75, 80, 85, 90; left whisker starts at about 50, box left edge (Q1) at about 57, median line at about 64, box right edge (Q3) at about 80, right whisker ends at about 86.]

# Question 1

Use the five-number summary from part A and the boxplot of Breed J to compare the center and variability for the distribution of weight for the Breed H goats and the distribution of weight for the Breed J goats, in context.

# Question 1

2026 ▪ Q1

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Weights of Breed H Goats (pounds)

48, 48, 55, 56, 56, 57, 62, 66, 72, 72, 72, 73, 80, 80

**(c)(i)** A stem-and-leaf plot is another way to display the weights of goats. The distribution of weight for the Breed H goats is displayed in the given stem-and-leaf plot.

Weights of Breed H Goats (pounds)

# Question 1

```
4 | 8 8
5 | 5 6 6 7
6 | 2 6
7 | 2 2 2 3
8 | 0 0
```

# Question 1

Key: $4|8 = 48$ pounds
Consider the five-number summary from part A and the stem-and-leaf plot.
If a boxplot were created from the five-number summary found in part A, what characteristic of the shape of the distribution of weight for the Breed H goats would be apparent from the stem-and-leaf plot but not from the boxplot?
**(c)(ii)** Explain why a boxplot would not display the characteristic of the shape of the distribution identified in part C (i).

# Question 1

2025 ▪ Q1

The manager of an automotive company is interested in comparing the gas mileages for cars manufactured in Country A and cars manufactured in Country B. The manager selected a random sample of 100 cars manufactured in Country A and a random sample of 100 cars manufactured in Country B. The gas mileages for each sample, in miles per gallon (mpg), are summarized in the boxplots.

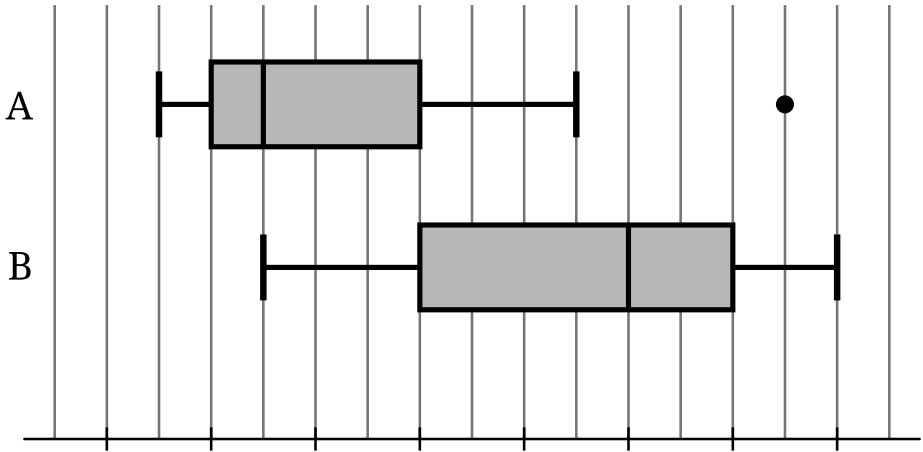
**Boxplots of Gas Mileage for Each Country**

[Boxplots comparing gas mileage (mpg) for Country A and Country B, drawn above a shared horizontal axis "Gas Mileage (mpg)" numbered 12 to 40 in increments of 4. Country A: left whisker to about 14, box (Q1) starts at about 16, median line at about 17, box (Q3) ends at about 24, right whisker to about 30, and an outlier point at about 38. Country B: left whisker to about 18, box (Q1) starts at about 24, median line at about 32, box (Q3) ends at about 36, right whisker to about 40.]

# Question 1

**(a)** Compare the distributions of gas mileage for the sample of cars manufactured in Country A and the sample of cars manufactured in Country B.
**(b)** For the distribution of gas mileage for the sample of cars manufactured in Country A, would you expect the mean to be greater than 18 mpg, less than 18 mpg, or equal to 18 mpg? Justify your answer.
**(c)(i)** The manager will create a new boxplot with the combined data from the sample of cars manufactured in Country A and the sample of cars manufactured in Country B. What is the range of the combined data set? Justify your answer.
**(c)(ii)** What is a possible value of the median of the combined data set? Justify your answer by referencing the boxplots shown.

# Question 1



# Solution 1

## (a) Mark scheme

- Country A's gas mileage distribution has a lower center than Country B's: median A = 18 mpg is less than median B = 32 mpg. Spread: the IQR of A (8 mpg) is less than the IQR of B (12 mpg), even though the range of A (24 mpg) is slightly greater than the range of B (22 mpg). The car in Country A with 38 mpg (the maximum of A) is an outlier; Country B's distribution has no outliers. Full credit (1 mark) requires at least 3 of these 4 components: (1) directly compares center for the two distributions, (2) directly compares spread (IQR or range) for the two distributions, (3) indicates Country A has an outlier, (4) sufficient context (both country names AND 'gas mileage'/'mpg'). Do not credit shape descriptions like

## Solution 1

'skewed'/'symmetric' toward these components (boxplots can't fully establish shape).

# Solution 1

**(b) Mark scheme**

- The mean is expected to be greater than 18 mpg. The median of Country A's gas-mileage distribution is 18 mpg. Because the distribution has an outlier to the right (is right-skewed, due to the 38 mpg car), the mean — which is not resistant to outliers — is pulled above the resistant median toward the higher values. Full credit needs: (1) states the mean is expected to be greater than 18 mpg, (2) states 18 as the median value, (3) a reasonable justification connecting the right-skew/outlier to mean $>$ median (a bare 'because the distribution is skewed' is too weak; an appropriate numerical argument also qualifies, but averaging portions of the five-number summary does not).

# Solution 1

## (c)(i) Mark scheme

- Range of the combined data set = 26 mpg. The maximum of the combined data is 40 mpg (Country B's maximum, since Country A's boxplot shows all its values below 40). The minimum of the combined data is 14 mpg (Country A's minimum, since Country B's boxplot shows all its values above 14). Range = 40 − 14 = 26 mpg. Full credit needs: (1) correctly calculates 26 mpg as the range, (2) justifies it by identifying 40 as the combined maximum and 14 as the combined minimum (a response that only restates 'ranges from 14 to 40' without computing 26 does not satisfy component 1 but may satisfy component 2).

# Solution 1

## (c)(ii) Mark scheme

- A possible median of the combined data set is 24 mpg. With 200 total values, the median is a value where at least 100 of the combined gas mileages are $\leq$ it and at least 100 are $\geq$ it. From Country A's boxplot, Q3 = 24 mpg, so at least 75 of A's 100 values are $\leq 24$. From Country B's boxplot, Q1 = 24 mpg, so at least 75 of B's 100 values are $\geq 24$ (and thus at least 25 of B's values are $\leq 24$, and at least 25 of A's values are $\geq 24$). Combining, at least 100 of the 200 combined values are $\leq 24$ and at least 100 are $\geq 24$, so 24 is a valid possible median. Full credit needs: (1) calculates 24 mpg as a possible median, (2) justifies it by showing at least half the combined values are $\leq 24$ and at least half are $\geq 24$ (referencing

## Solution 1

the boxplots' quartiles, a near-100th-value argument, or counting boxplot segments are all acceptable justifications).

# Question 6

2024 ▪ Q6

Begin your response to **QUESTION 6** on this page.

**SECTION II, Part B**

**Suggested Time—25 minutes**

**1 Question**

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. A company sells a certain type of whistle. The price of the whistle varies from store to store. Julio, a statistician at the company, wants to estimate the mean price, in dollars ($), of this type of whistle at all stores that sell the whistle.

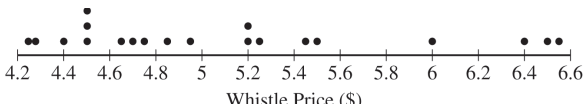
   (a) (i) Identify the appropriate inference procedure for Julio to use.

# Question 6

(ii) Describe the parameter for the inference procedure you identified in part (a-i) in context.

Julio called the managers of 20 randomly selected stores that sell the whistle and recorded the price of the whistle at each store. Following is a dotplot of Julio's data.

**Price of the Whistle at 20 Stores**



# Question 6

# Question 6

Continue your response to **QUESTION 6** on this page.

The summary statistics for Julio's data are shown in the following table.

# Question 6

(b) Julio wants to examine some characteristics of the distribution of the sample of whistle prices.

(i) Describe the shape of the distribution of the sample of whistle prices. Justify your response using appropriate values from the summary statistics table.

# Question 6

(ii) Using the $1.5 \times \text{IQR}$ rule, determine whether there are any outliers in the sample of whistle prices. Justify your response.

# Question 6

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 6** on this page.

It can often be difficult to determine whether the distribution of sample data is skewed by looking at a graph of the data and the summary statistics, particularly when the sample size is small. Thus, statisticians sometimes measure how skewed a data set is. One such measure is Pearson's coefficient of skewness, which is calculated using the following formula.

$$\text{Pearson's Coefficient of Skewness} = \frac{3(\overline{x} - m)}{s}$$

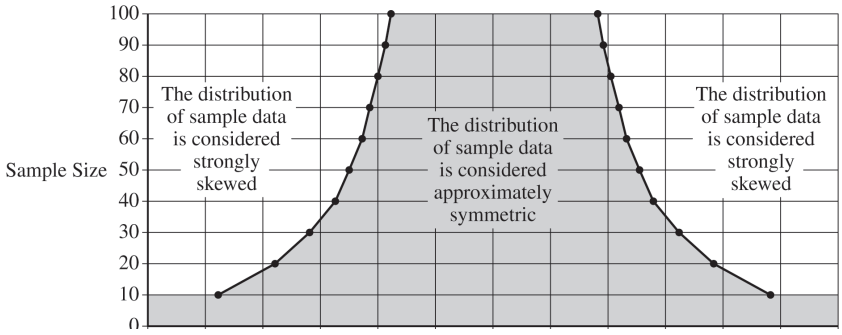
In the formula, $\overline{x}$ is the sample mean, $m$ is the sample median, and $s$ is the sample standard deviation.

(c) (i) Calculate Pearson's coefficient of skewness for Julio's sample of 20 whistle prices. Show your work.

# Question 6

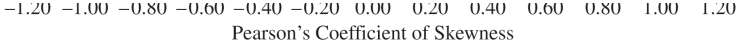
The following graph shows conclusions that can be made about the shape of the distribution of sample data based on Pearson's coefficient of skewness and sample size.

**Conclusion from Pearson's Coefficient of Skewness**



# Question 6

# Question 6




(ii) Indicate the value of the Pearson's coefficient of skewness you calculated in part (c-i) for the appropriate sample size by marking it with an "X" on the preceding graph.

# Question 6

Julio's inference procedure in part (a-i) needs one of the following requirements to be satisfied to verify the normality condition.

- The sample size is greater than or equal to 30.
- If the sample size is less than 30, the distribution of the sample data is not strongly skewed and does not have outliers.

(ii) Using your response to (d-i) and the preceding requirements, is the normality condition satisfied for Julio's data? Explain your response.

# Question 6

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# Question 6

**STOP**

**END OF EXAM**

# Question 6

# Question 6

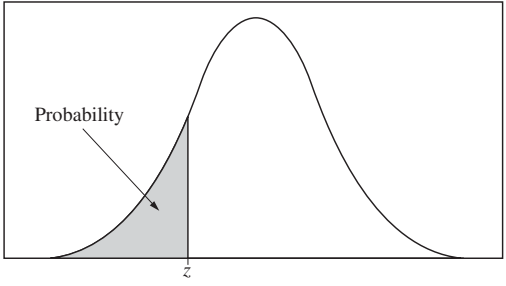



Table entry for $z$ is the probability lying below $z$.

# Question 6

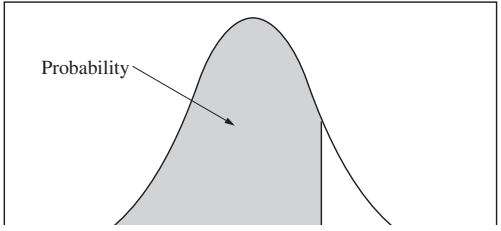
**Table A Standard normal probabilities**

| $z$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| −3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| −3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| −3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| −3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| −2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| −2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| −2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| −2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| −2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| −2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| −2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| −2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| −2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| −2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| −1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| −1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| −1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| −1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| −1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| −1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| −1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| −1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| −1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |

# Question 6

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| −1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| −1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| −0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| −0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| −0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| −0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| −0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| −0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| −0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| −0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| −0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| −0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

Table entry for $z$ is the



# Question 6

probability lying below $z$.


**Table A** (*Continued*)

| $z$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | 8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |

# Question 6

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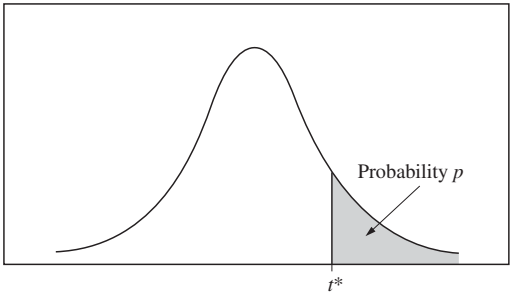
|---|---|---|---|---|---|---|---|---|---|---|
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |

# Question 6

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | 9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

Table entry for $p$ and $C$ is the point $t^*$ with probability $p$ lying above it and probability $C$ lying between $-t^*$ and $t^*$.



# Question 6

**Table B** ***t* distribution critical values**

| | Tail probability $p$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| df | .25 | .20 | .15 | .10 | .05 | .025 | .02 | .01 | .005 | .0025 | .001 | .0005 |
| 1 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.71 | 15.89 | 31.82 | 63.66 | 127.3 | 318.3 | 636.6 |
| 2 | .816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 4.849 | 6.965 | 9.925 | 14.09 | 22.33 | 31.60 |
| 3 | .765 | .978 | 1.250 | 1.638 | 2.353 | 3.182 | 3.482 | 4.541 | 5.841 | 7.453 | 10.21 | 12.92 |
| 4 | .741 | .941 | 1.190 | 1.533 | 2.132 | 2.776 | 2.999 | 3.747 | 4.604 | 5.598 | 7.173 | 8.610 |
| 5 | .727 | .920 | 1.156 | 1.476 | 2.015 | 2.571 | 2.757 | 3.365 | 4.032 | 4.773 | 5.893 | 6.869 |
| 6 | .718 | .906 | 1.134 | 1.440 | 1.943 | 2.447 | 2.612 | 3.143 | 3.707 | 4.317 | 5.208 | 5.959 |
| 7 | .711 | .896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.517 | 2.998 | 3.499 | 4.029 | 4.785 | 5.408 |
| 8 | .706 | .889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.449 | 2.896 | 3.355 | 3.833 | 4.501 | 5.041 |
| 9 | .703 | .883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.398 | 2.821 | 3.250 | 3.690 | 4.297 | 4.781 |
| 10 | .700 | .879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.359 | 2.764 | 3.169 | 3.581 | 4.144 | 4.587 |
| 11 | .697 | .876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.328 | 2.718 | 3.106 | 3.497 | 4.025 | 4.437 |
| 12 | .695 | .873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.303 | 2.681 | 3.055 | 3.428 | 3.930 | 4.318 |
| 13 | .694 | .870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.282 | 2.650 | 3.012 | 3.372 | 3.852 | 4.221 |
| 14 | .692 | .868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.264 | 2.624 | 2.977 | 3.326 | 3.787 | 4.140 |
| 15 | .691 | .866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.249 | 2.602 | 2.947 | 3.286 | 3.733 | 4.073 |
| 16 | .690 | .865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.235 | 2.583 | 2.921 | 3.252 | 3.686 | 4.015 |
| 17 | .689 | .863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.224 | 2.567 | 2.898 | 3.222 | 3.646 | 3.965 |
| 18 | .688 | .862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.214 | 2.552 | 2.878 | 3.197 | 3.611 | 3.922 |
| 19 | .688 | .861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.205 | 2.539 | 2.861 | 3.174 | 3.579 | 3.883 |
| 20 | .687 | .860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.197 | 2.528 | 2.845 | 3.153 | 3.552 | 3.850 |
| 21 | .686 | .859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.189 | 2.518 | 2.831 | 3.135 | 3.527 | 3.819 |
| 22 | .686 | .858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.183 | 2.508 | 2.819 | 3.119 | 3.505 | 3.792 |

# Question 6

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 23 | .685 | .858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.177 | 2.500 | 2.807 | 3.104 | 3.485 | 3.768 |
| 24 | .685 | .857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.172 | 2.492 | 2.797 | 3.091 | 3.467 | 3.745 |
| 25 | .684 | .856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.167 | 2.485 | 2.787 | 3.078 | 3.450 | 3.725 |
| 26 | .684 | .856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.162 | 2.479 | 2.779 | 3.067 | 3.435 | 3.707 |
| 27 | .684 | .855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.158 | 2.473 | 2.771 | 3.057 | 3.421 | 3.690 |
| 28 | .683 | .855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.154 | 2.467 | 2.763 | 3.047 | 3.408 | 3.674 |
| 29 | .683 | .854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.150 | 2.462 | 2.756 | 3.038 | 3.396 | 3.659 |
| 30 | .683 | .854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.147 | 2.457 | 2.750 | 3.030 | 3.385 | 3.646 |
| 40 | .681 | .851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.123 | 2.423 | 2.704 | 2.971 | 3.307 | 3.551 |
| 50 | .679 | .849 | 1.047 | 1.299 | 1.676 | 2.009 | 2.109 | 2.403 | 2.678 | 2.937 | 3.261 | 3.496 |
| 60 | .679 | .848 | 1.045 | 1.296 | 1.671 | 2.000 | 2.099 | 2.390 | 2.660 | 2.915 | 3.232 | 3.460 |
| 80 | .678 | .846 | 1.043 | 1.292 | 1.664 | 1.990 | 2.088 | 2.374 | 2.639 | 2.887 | 3.195 | 3.416 |
| 100 | .677 | .845 | 1.042 | 1.290 | 1.660 | 1.984 | 2.081 | 2.364 | 2.626 | 2.871 | 3.174 | 3.390 |
| 1000 | .675 | .842 | 1.037 | 1.282 | 1.646 | 1.962 | 2.056 | 2.330 | 2.581 | 2.813 | 3.098 | 3.300 |
| $\infty$ | .674 | .841 | 1.036 | 1.282 | 1.645 | 1.960 | 2.054 | 2.326 | 2.576 | 2.807 | 3.091 | 3.291 |
| | 50% | 60% | 70% | 80% | 90% | 95% | 96% | 98% | 99% | 99.5% | 99.8% | 99.9% |

Confidence level $C$

# Question 6

Table entry for $p$ is the point $(\chi^2)$ with probability $p$ lying above it.

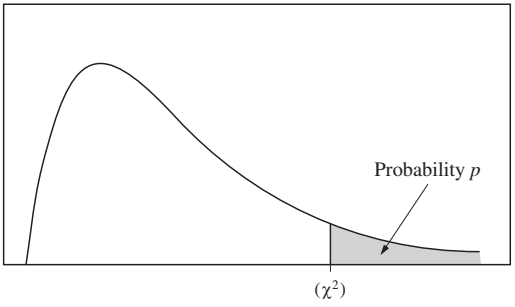



**Table C** $\chi^2$ **critical values**

# Question 6

| | Tail probability $p$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| df | .25 | .20 | .15 | .10 | .05 | .025 | .02 | .01 | .005 | .0025 | .001 | .0005 |
| 1 | 1.32 | 1.64 | 2.07 | 2.71 | 3.84 | 5.02 | 5.41 | 6.63 | 7.88 | 9.14 | 10.83 | 12.12 |
| 2 | 2.77 | 3.22 | 3.79 | 4.61 | 5.99 | 7.38 | 7.82 | 9.21 | 10.60 | 11.98 | 13.82 | 15.20 |
| 3 | 4.11 | 4.64 | 5.32 | 6.25 | 7.81 | 9.35 | 9.84 | 11.34 | 12.84 | 14.32 | 16.27 | 17.73 |
| 4 | 5.39 | 5.99 | 6.74 | 7.78 | 9.49 | 11.14 | 11.67 | 13.28 | 14.86 | 16.42 | 18.47 | 20.00 |
| 5 | 6.63 | 7.29 | 8.12 | 9.24 | 11.07 | 12.83 | 13.39 | 15.09 | 16.75 | 18.39 | 20.51 | 22.11 |
| 6 | 7.84 | 8.56 | 9.45 | 10.64 | 12.59 | 14.45 | 15.03 | 16.81 | 18.55 | 20.25 | 22.46 | 24.10 |
| 7 | 9.04 | 9.80 | 10.75 | 12.02 | 14.07 | 16.01 | 16.62 | 18.48 | 20.28 | 22.04 | 24.32 | 26.02 |
| 8 | 10.22 | 11.03 | 12.03 | 13.36 | 15.51 | 17.53 | 18.17 | 20.09 | 21.95 | 23.77 | 26.12 | 27.87 |
| 9 | 11.39 | 12.24 | 13.29 | 14.68 | 16.92 | 19.02 | 19.68 | 21.67 | 23.59 | 25.46 | 27.88 | 29.67 |
| 10 | 12.55 | 13.44 | 14.53 | 15.99 | 18.31 | 20.48 | 21.16 | 23.21 | 25.19 | 27.11 | 29.59 | 31.42 |
| 11 | 13.70 | 14.63 | 15.77 | 17.28 | 19.68 | 21.92 | 22.62 | 24.72 | 26.76 | 28.73 | 31.26 | 33.14 |
| 12 | 14.85 | 15.81 | 16.99 | 18.55 | 21.03 | 23.34 | 24.05 | 26.22 | 28.30 | 30.32 | 32.91 | 34.82 |
| 13 | 15.98 | 16.98 | 18.20 | 19.81 | 22.36 | 24.74 | 25.47 | 27.69 | 29.82 | 31.88 | 34.53 | 36.48 |
| 14 | 17.12 | 18.15 | 19.41 | 21.06 | 23.68 | 26.12 | 26.87 | 29.14 | 31.32 | 33.43 | 36.12 | 38.11 |
| 15 | 18.25 | 19.31 | 20.60 | 22.31 | 25.00 | 27.49 | 28.26 | 30.58 | 32.80 | 34.95 | 37.70 | 39.72 |
| 16 | 19.37 | 20.47 | 21.79 | 23.54 | 26.30 | 28.85 | 29.63 | 32.00 | 34.27 | 36.46 | 39.25 | 41.31 |
| 17 | 20.49 | 21.61 | 22.98 | 24.77 | 27.59 | 30.19 | 31.00 | 33.41 | 35.72 | 37.95 | 40.79 | 42.88 |
| 18 | 21.60 | 22.76 | 24.16 | 25.99 | 28.87 | 31.53 | 32.35 | 34.81 | 37.16 | 39.42 | 42.31 | 44.43 |
| 19 | 22.72 | 23.90 | 25.33 | 27.20 | 30.14 | 32.85 | 33.69 | 36.19 | 38.58 | 40.88 | 43.82 | 45.97 |
| 20 | 23.83 | 25.04 | 26.50 | 28.41 | 31.41 | 34.17 | 35.02 | 37.57 | 40.00 | 42.34 | 45.31 | 47.50 |
| 21 | 24.93 | 26.17 | 27.66 | 29.62 | 32.67 | 35.48 | 36.34 | 38.93 | 41.40 | 43.78 | 46.80 | 49.01 |
| 22 | 26.04 | 27.30 | 28.82 | 30.81 | 33.92 | 36.78 | 37.66 | 40.29 | 42.80 | 45.20 | 48.27 | 50.51 |
| 23 | 27.14 | 28.43 | 29.98 | 32.01 | 35.17 | 38.08 | 38.97 | 41.64 | 44.18 | 46.62 | 49.73 | 52.00 |
| 24 | 28.24 | 29.55 | 31.13 | 33.20 | 36.42 | 39.36 | 40.27 | 42.98 | 45.56 | 48.03 | 51.18 | 53.48 |

# Question 6

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 29.34 | 30.68 | 32.28 | 34.38 | 37.65 | 40.65 | 41.57 | 44.31 | 46.93 | 49.44 | 52.62 | 54.95 |
| 26 | 30.43 | 31.79 | 33.43 | 35.56 | 38.89 | 41.92 | 42.86 | 45.64 | 48.29 | 50.83 | 54.05 | 56.41 |
| 27 | 31.53 | 32.91 | 34.57 | 36.74 | 40.11 | 43.19 | 44.14 | 46.96 | 49.64 | 52.22 | 55.48 | 57.86 |
| 28 | 32.62 | 34.03 | 35.71 | 37.92 | 41.34 | 44.46 | 45.42 | 48.28 | 50.99 | 53.59 | 56.89 | 59.30 |
| 29 | 33.71 | 35.14 | 36.85 | 39.09 | 42.56 | 45.72 | 46.69 | 49.59 | 52.34 | 54.97 | 58.30 | 60.73 |
| 30 | 34.80 | 36.25 | 37.99 | 40.26 | 43.77 | 46.98 | 47.96 | 50.89 | 53.67 | 56.33 | 59.70 | 62.16 |
| 40 | 45.62 | 47.27 | 49.24 | 51.81 | 55.76 | 59.34 | 60.44 | 63.69 | 66.77 | 69.70 | 73.40 | 76.09 |
| 50 | 56.33 | 58.16 | 60.35 | 63.17 | 67.50 | 71.42 | 72.61 | 76.15 | 79.49 | 82.66 | 86.66 | 89.56 |
| 60 | 66.98 | 68.97 | 71.34 | 74.40 | 79.08 | 83.30 | 84.58 | 88.38 | 91.95 | 95.34 | 99.61 | 102.7 |
| 80 | 88.13 | 90.41 | 93.11 | 96.58 | 101.9 | 106.6 | 108.1 | 112.3 | 116.3 | 120.1 | 124.8 | 128.3 |
| 100 | 109.1 | 111.7 | 114.7 | 118.5 | 124.3 | 129.6 | 131.1 | 135.8 | 140.2 | 144.3 | 149.4 | 153.2 |

# Solution 6

**(a)(i)**

**Mark scheme**

- A one-sample $t$-interval for a population mean. Full credit requires identifying the procedure BY NAME as a one-sample $t$-interval (e.g. "one-sample $t$-interval"); a response that refers to a "test" instead of an interval does not satisfy this.

# Solution 6

**(a)(ii)**

**Mark scheme**

- The parameter is $\mu$ = the true mean price, in dollars, of this type of whistle at all stores that sell the whistle. Full credit needs the parameter identified as a MEAN (singular "a mean" is sufficient) AND sufficient context: the population (all stores that sell the whistle) and the variable of interest (the price of this type of whistle) – at minimum, "price" and "whistle" must appear, and if the response refers to the sample mean instead of the population mean it must be clearly defined as the population mean to earn credit.

# Solution 6

**(b)(i)**

**Mark scheme**

- The distribution of the sample of whistle prices appears slightly skewed to the right, because the mean ($5.12$) is slightly higher than the median ($4.885$). Full credit needs: (1) indicates the distribution is skewed right (or approximately symmetric, if justified consistently), and (2) justification using the summary statistics – e.g. the mean is higher than the median, the ratio of mean to median is greater than 1, or comparing the max-to-median gap with the median-to-min gap. A response that indicates skewed LEFT does not satisfy either component.

# Solution 6

**(b)(ii)**

**Mark scheme**

- Using the $1.5 \times IQR$ rule: $IQR = Q_3 - Q_1 = 5.475 - 4.51 = 0.965$. Lower boundary $= Q_1 - 1.5(IQR) = 4.51 - 1.5(0.965) = 3.0625$; because the minimum $(4.25)$ is greater than $3.0625$, there are no outliers to the left. Upper boundary $= Q_3 + 1.5(IQR) = 5.475 + 1.5(0.965) = 6.9225$; because the maximum $(6.58)$ is less than $6.9225$, there are no outliers to the right. There are NO outliers in this sample of whistle prices. Full credit needs both boundary values computed with work shown AND the correct no-outliers conclusion stated based on the calculated lower and upper criteria.

# Solution 6

**(c)(i)**

**Mark scheme**

- Pearson's coefficient of skewness $= \dfrac{3(\bar{x} - m)}{s} = \dfrac{3(5.12 - 4.885)}{0.743} \approx 0.949.$
  Full credit needs the correct value with work shown, substituting $\bar{x} = 5.12$ (mean), $m = 4.885$ (median), and $s = 0.743$ (standard deviation) into the given formula.

# Solution 6

**(c)(ii)**

**Mark scheme**

- Mark an "X" on the provided graph at the point where sample size $= 20$ (Julio's $n$) and Pearson's coefficient of skewness $\approx 0.949$ (from part (c-i)) – i.e. at approximately $(0.949, 20)$, which lies on the line for sample size 20, between about $0.90$ and $1.00$ on the horizontal axis, inside the right-hand shaded "strongly skewed" region near the boundary curve. Full credit needs the mark placed at (or consistent with) that coordinate.

# Solution 6

**(d)(i)**

**Mark scheme**

- The distribution of the sample of whistle prices should be considered STRONGLY skewed: for a sample size of 20 and a skewness coefficient of $0.949$ (from part (c)), that point falls in the "strongly skewed" region of the graph in part (c). Full credit needs the conclusion (strongly skewed) stated, using or referring to the graph or Pearson's coefficient of skewness from part (c) to make the determination. A response that indicates the distribution is strongly skewed to the LEFT does not satisfy this.

# Solution 6

## (d)(ii) Mark scheme

- No, the normality condition is NOT satisfied for Julio's data. Julio's sample size is only 20, which is less than 30, and (from part (d-i)) Pearson's coefficient of skewness indicates the distribution of the sample data is strongly skewed – so neither of the two allowed criteria (sample size $\geq 30$, OR sample size $< 30$ with the distribution not strongly skewed and no outliers) is met. Full credit needs: (1) states the normality condition is not met (or a statement consistent with part (d-i)), (2) explains that the sample size (20) is less than 30, (3) explains that the distribution of the sample data is strongly skewed, consistent with part (d-i). (A response that says the sample is strongly skewed but claims the normality condition IS satisfied should be scored as not meeting this.)

# Question 1

2021 ▪ Q1

The length of stay in a hospital after receiving a particular treatment is of interest to the patient, the hospital, and insurance providers. Of particular interest are unusually short or long lengths of stay. A random sample of 50 patients who received the treatment was selected, and the length of stay, in number of days, was recorded for each patient. The results are summarized in the following table and are shown in the dotplot.

| **Length of stay (days)** | **5** | **6** | **7** | **8** | **9** | **12** | **21** |
|---|---|---|---|---|---|---|---|
| Number of patients | 4 | 13 | 14 | 11 | 6 | 1 | 1 |

# Question 1

[Dotplot titled “Length of Stay (days)”; x-axis labeled Length of Stay (days), tick marks at 5, 7, 9, 11, 13, 15, 17, 19, 21. Stacked dots above each value show the counts from the table: 4 dots at 5, 13 dots at 6, 14 dots at 7, 11 dots at 8, 6 dots at 9, 1 dot at 12, 1 dot at 21 (no dots shown at 6, 10, 11, or 13-20 other than the isolated points at 12 and 21).]

**(a)** Determine the five-number summary of the distribution of length of stay.

**(b)(i)** Consider two rules for identifying outliers, method A and method B. Let method A represent the $1.5 \times$ IQR rule, and let method B represent the 2 standard deviations rule.

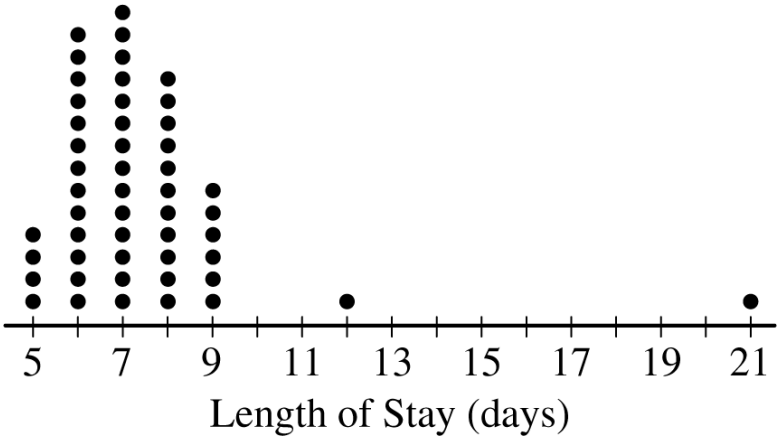
Using method A, determine any data points that are potential outliers in the distribution of length of stay. Justify your answer.

# Question 1

**(b)(ii)** The mean length of stay for the sample is $7.42$ days with a standard deviation of $2.37$ days. Using method B, determine any data points that are potential outliers in the distribution of length of stay. Justify your answer.

**(c)** Explain why method A might identify more data points as potential outliers than method B for a distribution that is strongly skewed to the right.

# Question 1



# Solution 1

**(a)**

**Mark scheme**

- Five-number summary of length of stay (days): Minimum $= 5$, Lower quartile $Q_1 = 6$, Median $= 7$, Upper quartile $Q_3 = 8$, Maximum $= 21$. Full credit requires all five values, each correctly labeled (min, $Q_1$, median, $Q_3$, max); 3-4 correct labeled values earns partial credit.

# Solution 1

**(b)(i)**

**Mark scheme**

- Method A ($1.5\times IQR rule$) :IQR $= Q_3 - Q_1 = 8 - 6 = 2$. Lower fence $= Q_1 - 1.5(IQR) = 6 - 1.5(2) = 3$; Upper fence $= Q_3 + 1.5(IQR) = 8 + 1.5(2) = 11$. Any value below 3 or above 11 is a potential outlier. No values are below 3; the patients who stayed 12 days and 21 days are above 11, so those two data points (12 and 21 days) are the potential outliers. Credit requires both fence values shown/calculated AND the two correct outliers identified.

# Solution 1

**(b)(ii)**

**Mark scheme**

- Method B (2 standard deviations rule): mean = 7.42 days, SD = 2.37 days. Mean $\pm 2 \times SD = 7.42 \pm 2(2.37) = 7.42 \pm 4.74$, giving the interval $(2.68, 12.16)$. Any value outside this interval is a potential outlier. Only the patient who stayed 21 days falls outside ($21 > 12.16$), so 21 days is the only outlier under method B. Credit requires the interval calculation shown AND the one correct outlier (21 days) identified.

# Solution 1

**(c) Mark scheme**

- For a distribution strongly skewed to the right, the mean is pulled toward the extreme values in the long right tail more than the median/quartiles are, and the ratio of SD to IQR tends to be larger than for a symmetric distribution. This means method B's outlier boundary (mean + $2\times SD$) $shifts further into the right tail than method A's boundary$ ($\mathrm{Q}_3 + 1.5 \times IQR$), so method B's threshold for calling a point an outlier becomes looser (higher). This decreases method B's ability to detect outliers relative to method A, so method A may flag more potential outliers than method B for a strongly right-skewed distribution. Credit requires (1) stating the mean/SD are pulled more toward the

## Solution 1

tail than the median/quartiles (or IQR), AND (2) linking that effect to method A detecting relatively more outliers than method B.

# Question 1

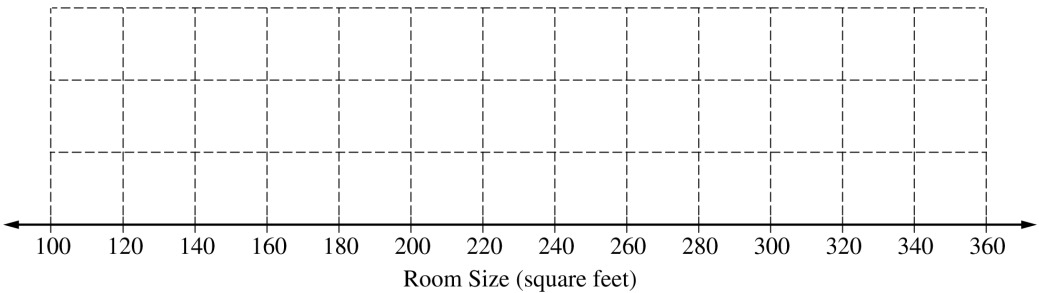
2019 ▪ Q1

The sizes, in square feet, of the 20 rooms in a student residence hall at a certain university are summarized in the following histogram.

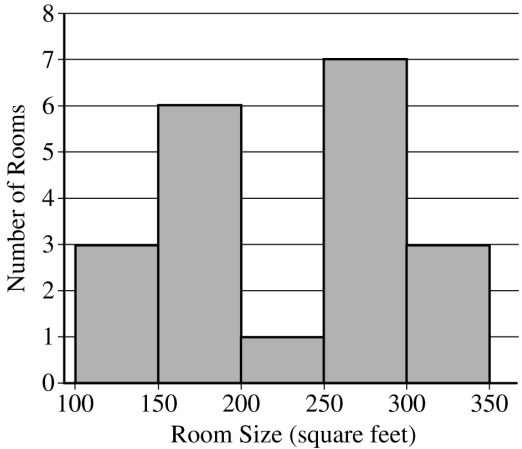
[Histogram titled "Room Size (square feet)"; y-axis "Number of Rooms" from 0 to 8; x-axis "Room Size (square feet)" with tick marks at 100, 150, 200, 250, 300, 350. Bars: 100-150 has height 3; 150-200 has height 6; 200-250 has height 1; 250-300 has height 7; 300-350 has height 3.]

**(a)** Based on the histogram, write a few sentences describing the distribution of room size in the residence hall.

# Question 1


100 120 140 160 180 200 220 240 260 280 300 320 340 360
Room Size (square feet)

# Question 1


Number of Rooms
0
1
2
3
4
5
6
7
8
100
150
200
250
300
350
Room Size (square feet)

# Question 1

| Mean | Standard Deviation | Min | Q1 | Median | Q3 | Max |
|---|---|---|---|---|---|---|
| 231.4 | 68.12 | 134 | 174 | 253.5 | 292 | 315 |

# Question 1

2019 ▪ Q1

The sizes, in square feet, of the 20 rooms in a student residence hall at a certain university are summarized in the following histogram.
[Histogram titled "Room Size (square feet)"; y-axis "Number of Rooms" from 0 to 8; x-axis "Room Size (square feet)" with tick marks at 100, 150, 200, 250, 300, 350. Bars: 100-150 has height 3; 150-200 has height 6; 200-250 has height 1; 250-300 has height 7; 300-350 has height 3.]

**(b)** Summary statistics for the sizes are given in the following table.

| Mean | Standard Deviation | Min | Q1 | Median | Q3 | Max |
|---|---|---|---|---|---|---|
| 231.4 | 68.12 | 134 | 174 | 253.5 | 292 | 315 |

# Question 1

Determine whether there are potential outliers in the data. Then use the following grid to sketch a boxplot of room size.

[Number line grid titled "Room Size (square feet)" with tick marks from 100 to 360 in increments of 20 (100, 120, 140, 160, 180, 200, 220, 240, 260, 280, 300, 320, 340, 360), for sketching a boxplot.]

# Question 1

2019 ▪ Q1

The sizes, in square feet, of the 20 rooms in a student residence hall at a certain university are summarized in the following histogram.

[Histogram titled "Room Size (square feet)"; y-axis "Number of Rooms" from 0 to 8; x-axis "Room Size (square feet)" with tick marks at 100, 150, 200, 250, 300, 350. Bars: 100-150 has height 3; 150-200 has height 6; 200-250 has height 1; 250-300 has height 7; 300-350 has height 3.]

**(c)** What characteristic of the shape of the distribution of room size is apparent from the histogram but not from the boxplot?

# Solution 1

## (a) Mark scheme

- The distribution of room sizes is bimodal and roughly symmetric, with two clusters: about 100-200 square feet and about 250-350 square feet. The center of the distribution is between 200 and 300 square feet (e.g., median $\approx 253.5$, mean $\approx 231.4$). The spread can be stated as: the range is a value between 150 and 250 square feet, OR the IQR is a value between 50 and 150 square feet, OR all room sizes are between 100 and 350 square feet. The response must include context (room size in square feet). Full credit requires all four components: (1) bimodal shape / two peaks / two clusters (NOT unimodal or normal), (2) center between 200 and 300 sq ft, (3) a valid spread statement, (4) context.

# Solution 1

**(b) Mark scheme**

- Outlier check: IQR = Q3 - Q1 = 292 - 174 = 118 square feet. Fences: $Q_1 - 1.5(IQR) = 174 - 1.5(118) = -3$ and $Q_3 + 1.5(IQR) = 292 + 1.5(118) = 469$. There are no potential outliers because the minimum (134 sq ft) does not fall below -3, and the maximum (315 sq ft) does not exceed 469. Boxplot: sketch a standard box-and-whisker (no mean shown) using the five-number summary min=134, Q1=174, median=253.5, Q3=292, max=315 – on the grid the minimum should fall between 120-140, Q1 between 160-180, the median between 240-260, Q3 between 280-300, and the maximum between 300-320 (square feet).

# Solution 1

**(c)**

**Mark scheme**

- The histogram clearly shows the bimodal nature of the distribution of room sizes (two distinct peaks/clusters), but this bimodal shape is not apparent from the boxplot – a boxplot only displays the five-number summary and cannot reveal multiple modes. (A response based only on symmetry/skewness, rather than the bimodal/unimodal shape, does not earn credit here.)

# Question 6

2019 ▪ Q6

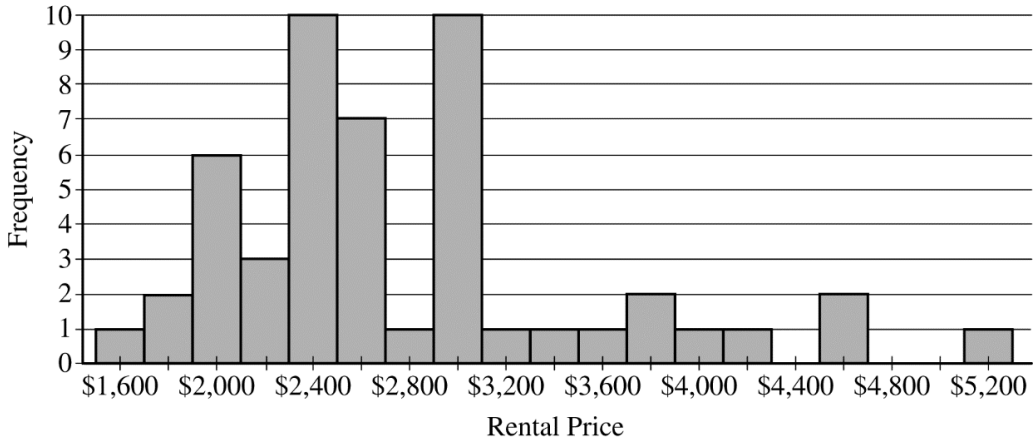
Emma is moving to a large city and is investigating typical monthly rental prices of available one-bedroom apartments. She obtained a random sample of rental prices for 50 one-bedroom apartments taken from a Web site where people voluntarily list available apartments.

**(a)** Describe the population for which it is appropriate for Emma to generalize the results from her sample.

# Question 6

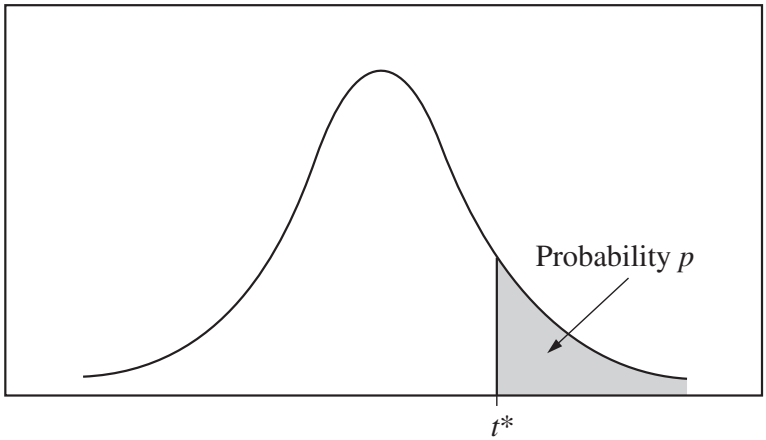
| Bootstrap Distribution of Medians | | | | | |
|---|---|---|---|---|---|
| Median | Frequency | Median | Frequency | Median | Frequency |
| 2,345 | 1 | 2,585 | 1 | 2,825 | 247 |
| 2,390 | 13 | 2,587.5 | 171 | 2,837.5 | 7 |
| 2,395 | 18 | 2,600 | 22 | 2,847.5 | 1 |
| 2,400 | 56 | 2,612.5 | 1,190 | 2,872.5 | 317 |
| 2,445 | 4 | 2,625 | 174 | 2,885 | 10 |
| 2,447.5 | 56 | 2,672.5 | 5 | 2,950 | 700 |
| 2,450 | 55 | 2,675 | 1,924 | 2,962.5 | 93 |
| 2,475 | 3 | 2,687.5 | 1,341 | 2,972.5 | 6 |
| 2,495 | 66 | 2,700 | 2,825 | 2,975 | 65 |
| 2,497.5 | 136 | 2,735 | 35 | 2,985 | 12 |
| 2,500 | 1,899 | 2,747.5 | 619 | 2,987.5 | 1 |
| 2,522.5 | 2 | 2,750 | 2 | 2,995 | 6 |
| 2,525 | 945 | 2,795 | 278 | 3,000 | 2 |
| 2,550 | 1,673 | 2,812.5 | 16 | 3,062.5 | 3 |

# Question 6


Frequency
10
9
8
7
6
5
4
3
2
1
0
$1,600
$2,000
$2,400
$2,800
$3,200
$3,600
$4,000
$4,400
$4,800
$5,200
Rental Price

# Question 6

Table entry for $p$ and $C$ is the point $t^*$ with probability $p$ lying above it and probability $C$ lying between $-t^*$ and $t^*$.



# Question 6

2019 ▪ Q6

Emma is moving to a large city and is investigating typical monthly rental prices of available one-bedroom apartments. She obtained a random sample of rental prices for 50 one-bedroom apartments taken from a Web site where people voluntarily list available apartments.

**(b)** The distribution of the 50 rental prices of the available apartments is shown in the following histogram.

[Histogram titled "Rental Price"; y-axis "Frequency" from 0 to 10; x-axis with tick marks at $1,600, $2,000, $2,400, $2,800, $3,200, $3,600, $4,000, $4,400, $4,800, $5,200.]

Emma wants to estimate the typical rental price of a one-bedroom apartment in the city. Based on the distribution shown, what is a disadvantage of using the mean rather than the median as an estimate of the typical rental price?

# Question 6

2019 ▪ Q6

Emma is moving to a large city and is investigating typical monthly rental prices of available one-bedroom apartments. She obtained a random sample of rental prices for 50 one-bedroom apartments taken from a Web site where people voluntarily list available apartments.

**(c)** Instead of using the sample median as the point estimate for the population mean, Emma wants to explore using the sampling distribution of the sample median for samples of size 50. Emma has one point, her sample median, that she can use to approximate the sampling distribution of the sample medians of size 50. Instead, Emma decides to use a theoretical sampling distribution to approximate the sampling distribution of the sample medians of size 50.

# Question 6

Because Emma does not have the resources to develop the theoretical sampling distribution, she estimates the sampling distribution of the sample median using a process called bootstrapping. In the bootstrapping process, a computer program performs the following steps.

- Take a random sample, with replacement, of size 50 from the original sample.
- Calculate and record the median of the sample.
- Repeat the process to obtain a total of 15,000 medians.

Emma ran the bootstrap process, and the following frequency table shows the results of generating 15,000 medians.

Bootstrap Distribution of Medians

| Median | Frequency | Median | Frequency | Median | Frequency |
|---|---|---|---|---|---|
| 2,345 | 3 | 2,585 | 1 | 2,825 | 247 |
| 2,390 | 13 | 2,587.5 | 171 | 2,837.5 | 7 |
| 2,395 | 2 | 2,600 | 22 | 2,847.5 | 1 |
| 2,400 | 56 | 2,612.5 | 1,190 | 2,872.5 | 317 |
| 2,445 | 4 | 2,625 | 174 | 2,885 | 10 |
| 2,447.5 | 56 | 2,672.5 | 5 | 2,950 | 700 |
| 2,450 | 55 | 2,675 | 1,924 | 2,962.5 | 93 |
| 2,475 | 3 | 2,687.5 | 1,341 | 2,972.5 | 6 |
| 2,495 | 66 | 2,700 | 2,825 | 2,975 | 65 |

# Question 6

The bootstrap distribution provides an approximation of the sampling distribution of the sample median. A confidence interval for the median can be constructed using a percentage of the values in the middle of the bootstrap distribution.

# Question 6

2019 ▪ Q6

Emma is moving to a large city and is investigating typical monthly rental prices of available one-bedroom apartments. She obtained a random sample of rental prices for 50 one-bedroom apartments taken from a Web site where people voluntarily list available apartments.

**(d)(i)** Use the frequency table to find the following.

Value of the 5th percentile.

**(d)(ii)** Use the frequency table to find the following.

Value of the 95th percentile.

# Question 6

2019 ▪ Q6

Emma is moving to a large city and is investigating typical monthly rental prices of available one-bedroom apartments. She obtained a random sample of rental prices for 50 one-bedroom apartments taken from a Web site where people voluntarily list available apartments.

**(e)** Find the percentage of bootstrap medians in the table that are equal to or between those found in part (d).

# Question 6

2019 ▪ Q6

Emma is moving to a large city and is investigating typical monthly rental prices of available one-bedroom apartments. She obtained a random sample of rental prices for 50 one-bedroom apartments taken from a Web site where people voluntarily list available apartments.

**(f)** Use your values from parts (d) and (e) to construct and interpret a confidence interval for the median rental price.

# Solution 6

**(a)**

**Mark scheme**

- Because random sampling was used, the results of the sample may be generalized to the population of rental prices for one-bedroom apartments in the city that are listed on this particular website at the time the sample was taken.

# Solution 6

**(b) Mark scheme**

- Because the distribution of the 50 rental prices in the sample is skewed to the right, the sample median provides a better indicator of typical rental prices than the sample mean. Some very large (high) rental prices pull the sample mean up, so the sample mean would overestimate the typical rental price, whereas the sample median would give a more accurate representation of a typical rental price. Must both (1) identify that using the mean overestimates the typical rental price, and (2) link this disadvantage to a feature of the distribution's shape (skewness) evident in the histogram – merely noting the mean is larger than the median is not by itself sufficient.

# Solution 6

**(c)**

**Mark scheme**

- To determine the sampling distribution of median rental prices for random samples of size 50 from this population, Emma would need to obtain every possible sample of 50 one-bedroom apartments from this population (i.e., from the website's full listing population) and compute the median of each sample. The collection of all of these possible sample medians is the theoretical sampling distribution of the sample median.

# Solution 6

**(d)(i)**

**Mark scheme**

- $(0.05)(15{,}000)=750$. The 5th percentile is a value $x_{0.05}$ such that at least 750 of the 15,000 bootstrap sample medians in the table are less than or equal to $x_{0.05}$. Cumulating frequencies from the smallest sample median in the table (going down the columns) until first reaching 750 values gives $x_{0.05}=\$2{,}500$.

# Solution 6

**(d)(ii)**

**Mark scheme**

- $(0.95)(15{,}000)=14{,}250$. Similarly, cumulating frequencies from the smallest value until reaching 14,250 (i.e., leaving 750 values above) gives the 95th percentile $x_{0.95}=\$2{,}950$.

# Solution 6

**(e)**

**Mark scheme**

- The percentage of the 15,000 bootstrap medians that are equal to or between the part-(d) values $(2,500 \textit{and} 2{,}950)$ is $\dfrac{14{,}404}{15{,}000} \times 100\% \approx 96.03\%$. (Any answer using plausible part-(d) values between 2,345 and 3,062.5, computed consistently, is acceptable.)

# Solution 6

## (f) Mark scheme

- Using the results from parts (d) and (e), an approximate 96 percent confidence interval for the median rental price of all one-bedroom apartments listed on this website for this city is $(2,500$,2,950). Interpretation: we are approximately 96 percent confident that the median rental price of all one-bedroom apartments listed on this website for this city is between $2,500 and 2{,}950$. Full credit requires: (1) using $2,500 and 2{,}950$ (the part-(d) percentile values) as the interval endpoints, (2) stating an approximate 90% or 96% confidence level (both accepted, consistent with part (e)), (3) a correct statement in context that the interval is specifically for the median (not the mean) rental price.

# Question 5

2018 ▪ Q5

The following histograms summarize the teaching year for the teachers at two high schools, A and B.

[Histogram "High School A": x-axis "Teaching Year" labeled at 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34 (each bar spans a 3-year interval, left endpoint inclusive); y-axis "Frequency" from 0 to 80 in increments of 10. Approximate bar heights by interval start: 1→46, 4→48, 7→24, 10→45, 13→29, 16→4, 19→1, 22→0, 25→1, 28→1, 31→1, 34→0.]

[Histogram "High School B": x-axis "Teaching Year" labeled at 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34 (each bar spans a 3-year interval, left endpoint inclusive); y-axis "Frequency" from 0 to 80 in increments of 10. Approximate bar heights by interval start: 1→79, 4→34, 7→28, 10→29, 13→19, 16→12, 19→8, 22→3, 25→4, 28→2, 31→3, 34→0.]

# Question 5

Teaching year is recorded as an integer, with first-year teachers recorded as 1, second-year teachers recorded as 2, and so on. Both sets of data have a mean teaching year of 8.2, with data recorded from 200 teachers at High School A and 221 teachers at High School B. On the histograms, each interval represents possible integer values from the left endpoint up to but not including the right endpoint.

**(a)** The median teaching year for one high school is 6, and the median teaching year for the other high school is 7. Identify which high school has each median and justify your answer.

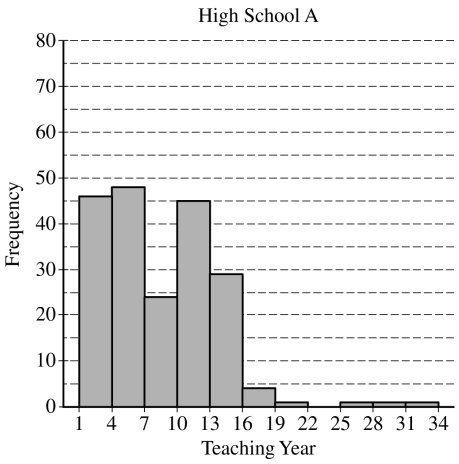
**(b)** An additional 18 teachers were not included with the data recorded from the 200 teachers at High School A. The mean teaching year of the 18 teachers is 2.5. What is the mean teaching year for all 218 teachers at High School A?

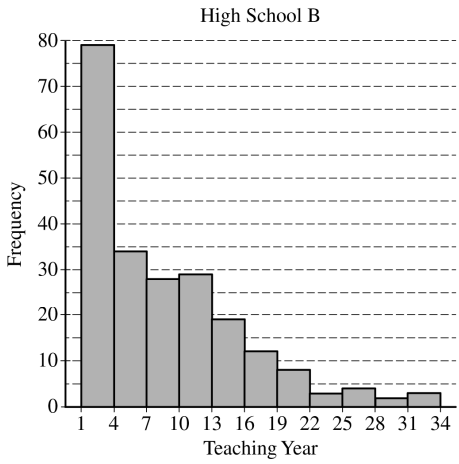
**(c)** The standard deviation of the teaching year for the 221 teachers at High School B is 7.2. If one teacher is selected at random from High School B, what is the probability

# Question 5

that the teaching year for the selected teacher will be within 1 standard deviation of the mean of $8.2$ ? Justify your answer.

# Question 5


High School A
Frequency
80
70
60
50
40
30
20
10
0
1 4 7 10 13 16 19 22 25 28 31 34
Teaching Year


High School B
Frequency
80
70
60
50
40
30
20
10
0
1 4 7 10 13 16 19 22 25 28 31 34
Teaching Year

# Solution 5

## (a) Mark scheme

- High School A has median teaching year 7; High School B has median teaching year 6. Reasoning: High School A's median needs 100 of its 200 values at or below it and 100 at or above; only 94 of A's values are below 7 while 106 are at least 7, so A's median lies in the interval [7,10) and cannot be less than 7. High School B's median is the 111th of 221 ordered values; more than half of B's values (113) are less than 7, so B's median lies in [4,7) and must be less than 7 — hence 6. (Equivalent argument: High School B's distribution is strongly right-skewed while A's is bimodal with a few high outliers; a strongly right-skewed distribution tends to have a mean substantially larger than its median, and since both schools have the same mean (8.2), the more right-skewed distribution, B, must be the one

## Solution 5

with the larger mean-to-median gap — i.e., the smaller median, 6.) Full credit requires: (1) stating median $= 7$ for School A and median $= 6$ for School B, (2) a reasonable explanation of how the decision was made, and (3) either explicitly using/applying the definition of median as the criterion, OR stating that School B is more skewed and explaining why the more-skewed distribution (B) has the larger mean-median separation. Describing either distribution as left-skewed, normal, or approximately normal lowers an otherwise-complete response by one level.

# Solution 5

## (b) Mark scheme

- Combined (weighted-average) mean for all 218 teachers at High School A: $\frac{(200)(8.2)+(18)(2.5)}{200+18} = \frac{1640+45}{218} = \frac{1685}{218} \approx 7.73$ years. Full credit requires both: (1) the correct answer, 7.73, and (2) enough work shown that the answer was obtained as a weighted average of the two individual group means (200 teachers at 8.2 years and 18 teachers at 2.5 years), not a simple unweighted average of 8.2 and 2.5.

# Solution 5

## (c) Mark scheme

- The interval mean $\pm$ 1 standard deviation is $8.2 \pm 7.2$, i.e. from 1.0 to 15.4 years. Since teaching year is recorded as an integer, this interval covers teaching years 1 through 15. Summing the heights of the five histogram bars covering that range (bins 1–4, 4–7, 7–10, 10–13, 13–16): $79 + 34 + 28 + 29 + 19 = 189$. Probability $= 189/221 \approx 0.8552$. Full credit requires: (1) correctly determining the appropriate interval is 1 to 15.4 (or 1 to 15 teaching years, or recognizing that all histogram bins up to 16 fall within one SD of the mean), (2) correctly summing the counts of data values in that interval based on the histogram (a count that is slightly off due to difficulty reading exact bar heights is still acceptable), and (3) computing the probability using 221 as the denominator. Using the Empirical Rule

## Solution 5

or a normal-distribution approximation instead of the actual finite population/histogram counts is NOT acceptable and earns no credit for this part.

# Question 1

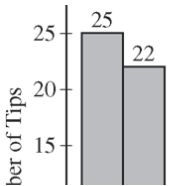
2016 ▪ Q1

**STATISTICS**
**SECTION II**
**Part A**
**Questions 1-5**
**Spend about 65 minutes on this part of the exam.**
**Percent of Section II score—75**

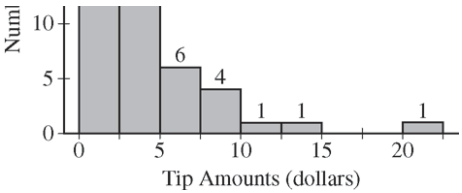
**Directions:** Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

# Question 1

1. Robin works as a server in a small restaurant, where she can earn a tip (extra money) from each customer she serves. The histogram below shows the distribution of her 60 tip amounts for one day of work.



# Question 1




(a) Write a few sentences to describe the distribution of tip amounts for the day shown.

(b) One of the tip amounts was $8. If the $8 tip had been $18, what effect would the increase have had on the following statistics? Justify your answers.

The mean:

The median:

# Solution 1

## (a) Mark scheme

- The distribution of Robin's tip amounts is skewed to the right, with a gap between the largest tip (in the $20-22.50$ interval) and the next-largest tip (in the $12.50-15$ interval) – the largest tip appears to be an outlier. The median tip is between $2.50 and 5.00$ (the mean is between $2.62 and 5.13$). Tip amounts vary from a minimum between $0 and 2.50$ up to a maximum between $20.00 and 22.50$; about 78% of tips are between $0 and 5$. Full credit (E) requires reasonable comments on all five components – shape, outlier/gap, center, variability, and context (tip amounts in dollars); partial credit (P) for 3-4 of the five; incorrect (I) for at most 2.

# Solution 1

**(b)**

**Mark scheme**

- Mean: if the \$8 tip had been \$18, the mean would increase by \$10 divided by 60 tips, i.e. by \$\tfrac{1}{6} \approx \$0.17\$, since the total sum of tips increases by \$10 while the number of tips (n = 60) stays the same. Median: the median would NOT change, because the current median lies between \$2.50 and \$5.00, and both \$8 and \$18 are above that value, so the tip's position among the ordered values doesn't shift the middle observation(s). Full credit requires all four components: states mean increases, correctly justifies why, states median unchanged, correctly justifies why.

# Question 4

2014 ▪ Q4

As part of its twenty-fifth reunion celebration, the class of 1988 (students who graduated in 1988) at a state university held a reception on campus. In an informal survey, the director of alumni development asked 50 of the attendees about their incomes. The director computed the mean income of the 50 attendees to be $189,952. In a news release, the director announced, "The members of our class of 1988 enjoyed resounding success. Last year's mean income of its members was $189,952!"

**(a)** What would be a statistical advantage of using the median of the reported incomes, rather than the mean, as the estimate of the typical income?

# Question 4

2014 ▪ Q4

As part of its twenty-fifth reunion celebration, the class of 1988 (students who graduated in 1988) at a state university held a reception on campus. In an informal survey, the director of alumni development asked 50 of the attendees about their incomes. The director computed the mean income of the 50 attendees to be $189,952. In a news release, the director announced, "The members of our class of 1988 enjoyed resounding success. Last year's mean income of its members was $189,952!"

**(b)** The director felt the members who attended the reception may be different from the class as a whole. A more detailed survey of the class was planned to find a better estimate of the income as well as other facts about the alumni. The staff developed two methods based on the available funds to carry out the survey.

Method 1: Send out an e-mail to all 6,826 members of the class asking them to complete an online form. The staff estimates that at least 600 members will respond.

# Question 4

Method 2: Select a simple random sample of members of the class and contact the selected members directly by phone. Follow up to ensure that all responses are obtained. Because method 2 will require more time than method 1, the staff estimates that only 100 members of the class could be contacted using method 2.
Which of the two methods would you select for estimating the average yearly income of all 6,826 members of the class of 1988? Explain your reasoning by comparing the two methods and the effect of each method on the estimate.

# Solution 4

## (a) Mark scheme

- (Q4 is graded holistically across three official sections — section 1 = part (a), section 2 = part (b) component 1, section 3 = part (b) component 2 — via a combination table into an overall 0-4 score; this leaf covers section 1 only, so it carries about 1 of the 4 total marks.) The median is less affected by skewness and outliers than the mean. With a variable such as income, a small number of very large incomes could dramatically increase the mean but not the median, so the median would provide a better estimate of a typical income value. Full credit needs two components: (1) describes how skewness or outliers affect the mean but not (or less than) the median (e.g. 'the mean is affected by skewness/outliers,' 'the median is not affected by skewness/outliers,' or 'the mean

## Solution 4

is greater/less than the median when there is right/left skewness or outliers'), (2) makes a conjecture about a relevant characteristic of the distribution of incomes, such as skewness or an outlier. Partial credit for only one of the two components. Component (1) is NOT satisfied by unexplained claims such as 'don't use the mean for skewed distributions,' 'use the median for skewed distributions,' or an incorrect statement about means/medians (e.g. claiming the median is higher than the mean for right-skewed distributions). A single sentence can satisfy both components (e.g. 'If there was a billionaire in the sample, the mean would be higher than the median'). If a response instead argues that the mean is the more appropriate estimate of typical income, this leaf is scored one level down from what it otherwise would earn.

# Solution 4

**(b) Mark scheme**

- (This leaf covers sections 2 and 3 of Q4's three-section holistic scoring — about 3 of the question's 4 total marks.) Method 2 is better than Method 1. A sample obtained via Method 1 (e-mail everyone, ~600 respond) could be biased because of the voluntary nature of the response — it is plausible that class members with larger incomes would be more likely to respond, so the Method-1 sample mean would tend to overestimate the mean income of all class members. With Method 2 (a smaller random sample with follow-up), despite the smaller sample size, the random selection is likely to produce a sample that is more representative of the entire class, giving an unbiased estimate of the class's mean income — but if some of the members randomly selected for Method 2 do not respond, the incomes of

## Solution 4

responders may differ from the incomes of non-responders (e.g. if members with larger incomes are more or less willing to respond), and this nonresponse bias may produce a misleading estimate/conclusion about the mean income, including a direction consistent with whichever bias is identified (e.g. 'the sample mean is likely to be higher than the mean of the population'). Full credit needs, for section 2: chooses Method 2 AND identifies a relevant characteristic of Method 1 (its voluntary/self-selected nature is likely biased) AND a relevant characteristic of Method 2 (uses random selection, so is more representative) — responses that compare the two methods, or that only discuss negative characteristics of Method 1 and positive characteristics of Method 2, can satisfy both components even without explicitly stating 'Method 2'; and for section 3: indicates the incomes of responders may differ from the incomes of non-responders AND indicates the biased sampling method may produce a misleading estimate/conclusion about the

## Solution 4

mean income, including direction. A single sentence can satisfy both the first component of section 2 and the first component of section 3 (e.g. 'In Method 1, rich people are more likely to respond.'). Responses that focus on the larger sample size of Method 1 can still satisfy the section-3 direction component if they describe the effect as reducing the variability of the estimate; responses focused on untruthful survey answers can satisfy it if the effect on the estimate is stated appropriately. Discussions of conditions for inference are extraneous and should be ignored. Partial credit is earned section-by-section for providing only one of a section's two components (or, for section 2, for choosing Method 1 outright, which cannot earn credit for that section); this leaf's overall credit should reflect the AP holistic combination of sections 2 and 3.