Exercise walk-through

The Normal Distribution ▪ 1.10

eXercise

You must be able to

- describe the **normal distribution 正态分布** as a symmetric, bell-shaped model with mean μ and standard deviation σ
- apply the **empirical rule 经验法则** (68–95–99.7)
- calculate a z-**score** z **分数** and use the standard normal distribution

Method — The normal distribution

Symmetric and bell-shaped, fully described by its **mean** μ (center) and **standard deviation** σ (spread).

Method — The empirical rule

For a normal distribution, about **68%** of values lie within 1σ, **95%** within 2σ, and **99.7%** within 3σ of the mean.

Method — The z-score

$$z = \frac{x - \mu}{\sigma}$$

gives how many standard deviations x is from the mean. With $\mu = 100$, $\sigma = 15$, $x = 130$: $z = 2$.

Practice 2.1 ▪ 2 marks

State the three percentages of the empirical rule.

Solution 2.1

Method: The empirical rule

Worked steps

about 68% within 1σ, 95% within 2σ, 99.7% within 3σ B1 B1 *for two; for all three.*

Practice 2.2 ▪ 2 marks

A normal distribution has $\mu = 50$, $\sigma = 5$. Find the z-score of $x = 60$.

Solution 2.2

Worked steps

$$z = \frac{60 - 50}{5} = 2 \quad \text{M1}\ \text{A1}.$$

Practice 2.3 ▪ 1 mark

State the two parameters that define a normal distribution.

Solution 2.3

Worked steps

the mean μ and the standard deviation σ B1.

Exam-style 3.1 ▪ 1 mark

The empirical rule says about 95% of data lie within standard deviations of the mean.

A 1

B 2

C 3

D 4

Solution 3.1

Method: The empirical rule

A 1

B 2 ✓

C 3

D 4

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

A z-score measures

A the mean

B how many standard deviations a value is from the mean

C the median

D the range

Solution 3.2

A the mean

B how many standard deviations a value is from the mean ✓

C the median

D the range

Worked steps

B.

Exam-style 3.3 ▪ 2 marks

IQ scores are normal with $\mu = 100$ and $\sigma = 15$.
(a) Find the z-score for $x = 115$.
(b) State the percentage of scores within 1 standard deviation of the mean.

Solution 3.3

Method: The normal distribution

Worked steps

(a) $z = \dfrac{115 - 100}{15} = 1$ M1 A1. (b) about 68% B1.