

Chi-Square Tests

AP Statistics ■ Unit 8

AP Statistics



What we will cover

- 1 The chi-square idea
- 2 Goodness-of-fit test
- 3 Two-way tables
- 4 Homogeneity vs independence
- 5 Choosing a procedure





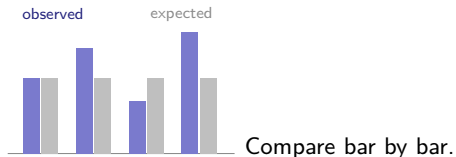
The chi-square idea

The **chi-square** tests compare counts:

- **observed** – what happened
- **expected** – what the null predicts

Big gaps $\Rightarrow$ doubt the claim.

Goodness-of-fit checks one variable;
a **two-way table** checks two.





Goodness-of-fit test

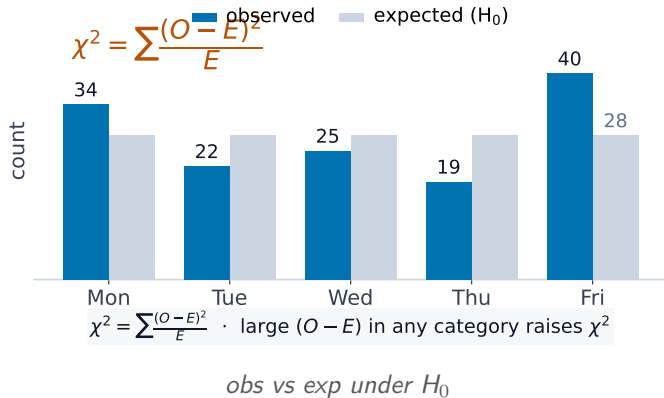
Does one variable follow a claimed distribution?

$$\chi^2 = \sum \frac{(\text{obs} - \text{exp})^2}{\text{exp}}$$

- $\text{expected} = \text{total} \times \text{claimed proportion}$
- condition: every expected ≥ 5
- $df = \text{categories} - 1$

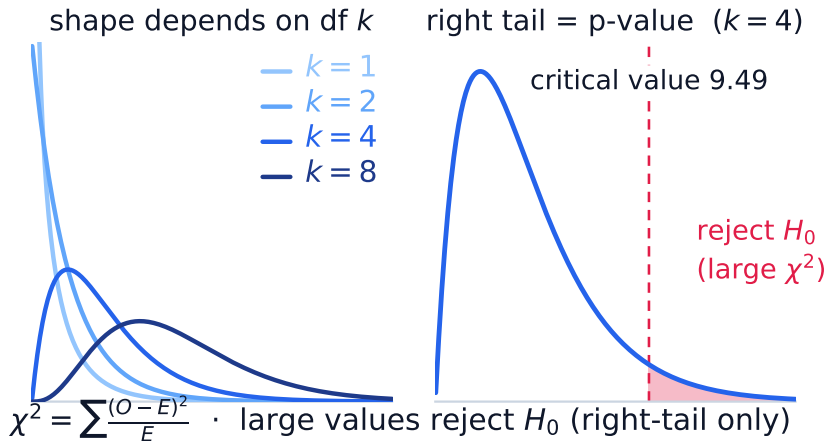
Large $\chi^2 \Rightarrow$ big gaps; the p-value is the **right**-tail area.

Goodness-of-fit test, in detail



Does one variable follow a claimed distribution?

Setting Up a Goodness-of-Fit Test



The chi-square distribution is right-skewed. A large statistic lands in the shaded right tail past the critical value – that is where you reject the model.

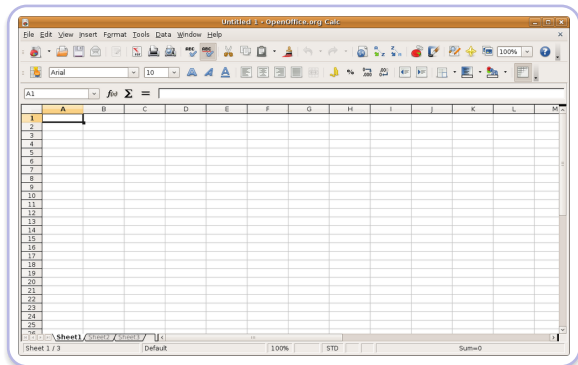


Two-way tables

For a two-way table, compute each **expected count** from the margins (under “no relationship”):

$$\text{exp} = \frac{(\text{row total})(\text{column total})}{\text{grand total}}$$

- every expected ≥ 5
- $df = (\text{rows} - 1)(\text{cols} - 1)$



Organise categorical counts before the test



Homogeneity vs independence

Same χ^2 mechanics – the *design* sets the name:

Homogeneity

several separate samples, one variable

H_0 : same distribution across groups

Independence

one sample, two variables recorded

H_0 : the two variables are unrelated

Separately survey men and women = homogeneity; one sample recording gender & preference = independence.

Three chi-square tests, one calculation

the shared arithmetic

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

in every case – only the question and the df change

goodness-of-fit

one categorical variable against a claimed distribution. $df = k - 1$

test for homogeneity

several populations, one variable – are the distributions the same?
Data from separate samples

test for independence

one population, two variables – are they associated? Data from one sample

the two-way df

$(\text{rows} - 1)(\text{columns} - 1)$ for both

the condition

every expected count at least 5

Homogeneity and independence use identical arithmetic and answer different questions. Which you have is decided by **how the data were collected**, not by the table.

The three chi-square tests

Goodness-of-fit

The **goodness-of-fit (GOF)** test: one categorical variable against a claimed distribution. Expected counts come from the claim.

Homogeneity

Several populations, one variable: are the distributions the same across the groups?

Independence

One population, two variables: are they associated? Expected counts are $\frac{\text{row total} \times \text{column total}}{n}$.

Choosing

Count the samples and the variables. One sample, one variable is GOF; several samples is homogeneity; one sample, two variables is independence.

All three use the same statistic and the same conditions – expected counts at least 5. Only the degrees of freedom and the conclusion differ.



Choosing a procedure

Count the samples and variables:

One proportion: one-sample z

Two proportions: two-sample z

Goodness-of-fit: one variable vs a claim

Homogeneity: several samples, one variable

Independence: one sample, two variables

Number of samples + number of variables point straight to the test.

Exam tips

- Use $\chi^2 = \sum \frac{(O-E)^2}{E}$ for **categorical** data; always divide by the **expected** count.
- Pick the right test: goodness-of-fit (one variable), independence, or homogeneity (two-way table).
- Compute expected counts as $\frac{\text{row total} \times \text{column total}}{\text{grand total}}$ and check each is ≥ 5 .
- A large χ^2 (small p-value) means observed counts differ from expected by more than chance.
- State degrees of freedom correctly (categories - 1, or $(r - 1)(c - 1)$).

Are My Results Unexpected?

When data are **counts** spread across several categories, we test whether the observed counts differ from what a claim predicts.

The tool is the **chi-square** (χ^2) statistic, which adds up the standardized gaps between observed and expected counts:

$$\chi^2 = \sum \frac{(O - E)^2}{E}.$$



Worked example – chi-square goodness of fit

A die is rolled 100 times: four faces give 30, 20, 25, 25 over four categories. Test whether it is fair.

- H_0 : all four categories are equally likely $\Rightarrow$ expected = $100/4 = 25$ each.
- $\chi^2 = \sum \frac{(O - E)^2}{E} = \frac{25}{25} + \frac{25}{25} + 0 + 0 = 2.0$
- df = categories - 1 = 3; critical value at $\alpha = 0.05$ is 7.81.
- $2.0 < 7.81 \Rightarrow$ fail to reject H_0 : no evidence the die is unfair.

Use **counts**, never percentages, and check every expected count is ≥ 5 .
“Fail to reject” is **not** “proved fair” – you just lack evidence against it.

Unit 8 – key takeaways

1

Idea

compare observed to expected counts; right-tail p-value

2

Statistic

$\chi^2 = \sum (O - E)^2 / E$; every expected ≥ 5

3

Tables

expected = row $\times$ col / grand; $df = (r - 1)(c - 1)$

4

Which test

fit, homogeneity, independence by design

Check yourself

- 1 Write the chi-square statistic formula.
- 2 What is the df for a goodness-of-fit with 6 categories?
- 3 How do you find an expected count in a two-way table?
- 4 Several separate samples, one variable: which test?



Answers 1. $\chi^2 = \sum (O - E)^2 / E$ 2. $6 - 1 = 5$ 3. row $\times$ col / grand total 4. homogeneity