

Inference for Proportions

AP Statistics ■ Unit 6

AP Statistics



What we will cover

- 1 Confidence interval for p
- 2 Interpreting an interval
- 3 Setting up a test
- 4 p-values and conclusions
- 5 Type I and II errors
- 6 Comparing two proportions





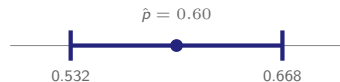
Confidence interval for a proportion

A **confidence interval** is estimate $\pm$ margin:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

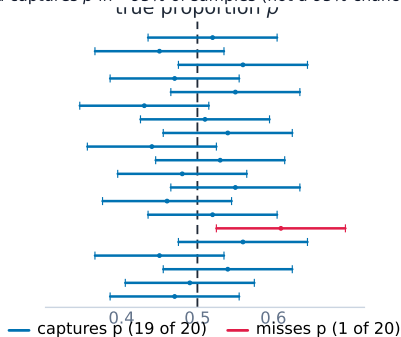
- **margin of error** = $z^* \cdot \text{standard error}$
- for 95%: **critical value** $z^* = 1.96$

Check: random, $\leq 10\%$ of population, and large counts.



Over many samples

95% CI: the method captures p in ~95% of samples (not a 95% chance that this one does)



Over many samples, about 95% of 95% confidence intervals capture the true proportion

Worked example – choosing the sample size

How many people must be surveyed for a 95% interval with margin of error at most 0.03?

- $z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \leq m$, solved for n
- use $\hat{p} = 0.5$ (the **worst case**, which maximises the product) and $z^* = 1.96$
- $n \geq \left(\frac{1.96}{0.03}\right)^2 (0.25) = 1067.1$
- round **up**: $n = 1068$

Two things are always marked: using $\hat{p} = 0.5$ when no estimate is given, and **rounding up** – rounding down would leave the margin too wide.

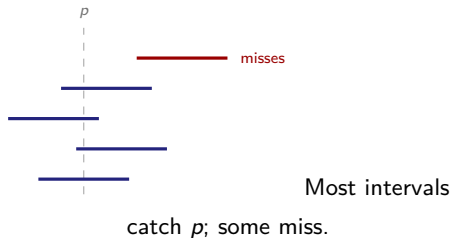


Interpreting an interval

Two layers of meaning:

- *interval*: “95% confident the true p is in $(0.532, 0.668)$ ”
- *level*: “95% of such intervals capture p ” – confidence is in the **method**

A value outside the interval is not plausible. Bigger $n \Rightarrow$ narrower; higher confidence $\Rightarrow$ wider.



Judging a claim from an interval

the test

if the claimed value lies **inside** the interval, the data are **consistent** with it

if outside

the data give **evidence against** the claim

what it does not say

an interval containing the value does not *prove* it – it fails to rule it out

the correct interpretation

“we are 95% confident that the true proportion lies between ... and ...”

what 95% refers to

the **method**: 95% of intervals built this way capture the true value. It is not a probability about this one interval

Never say “there is a 95% chance the parameter is in this interval”. The parameter is fixed; it is the **interval** that varies from sample to sample.



Setting up a test

A **significance test** weighs evidence against a claim.

- **null** $H_0 : p = p_0$ (“no effect”)
- **alternative** $H_a : p >, <, \text{ or } \neq p_0$

The **test statistic** (one-sample z-test), using p_0 :

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$$

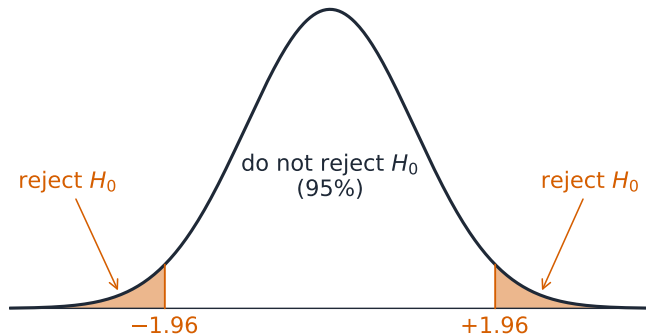
Check conditions with the **assumed** p_0 , not $\hat{p}$.

A coin

58 heads in 100: $\hat{p} = 0.58$, $H_0 : p = 0.5$, $H_a : p > 0.5$.

$$z = \frac{0.58 - 0.50}{0.05} = 1.6$$

Setting Up a Test for a Proportion



A two-tailed 5% test rejects the null hypothesis in the shaded tails

Worked example – a one-proportion z-test

A company claims 90% satisfaction. A sample of 100 finds 84 satisfied. Test $H_0 : p = 0.90$ against $H_a : p < 0.90$.

- $\hat{p} = 0.84$; use p_0 in the standard error, not $\hat{p}$
- $SE = \sqrt{\frac{0.90(0.10)}{100}} = 0.03$
- $z = \frac{0.84 - 0.90}{0.03} = -2.0$, so $p \approx 0.023$
- $0.023 < 0.05$: **reject H_0**

A *test* uses the **claimed** p_0 in the standard error; an *interval* uses $\hat{p}$. Mixing them is the commonest error here.

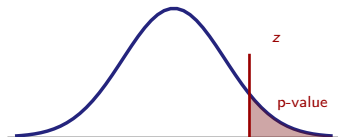


p-values and conclusions

The **p-value** p : *if H_0 were true*, the chance of a result at least this **extreme** – a tail area from z .

- $p \leq \alpha$: **reject** H_0
- $p > \alpha$: **fail to reject** H_0

Failing to reject is **not** proof of H_0 – just too little evidence.





Type I and Type II errors

A test on chance data can err two ways:

- **Type I** : reject a *true* H_0 (false alarm) – prob. α
- **Type II** : miss a *false* H_0 – prob. β

Power = $1 - \beta$ rises with larger n , a bigger true effect, or larger α .

	H_0 true	H_0 false
reject	Type I (α)	correct (power)
fail	correct	Type II (β)

The two errors, and writing the conclusion

the significance level

α – fixed **before** the data are seen; usually 0.05

the alternative hypothesis

H_a – one-sided or two-sided, chosen from the question, never from the data

a Type I error

rejecting a **true** H_0 – a false alarm. Its probability is α

a Type II error

failing to reject a **false** H_0 – a missed detection. Its probability is β

power

$1 - \beta$: the chance of detecting a real effect. Raised by a larger sample or a larger α

two proportions

use the **combined (pooled)** estimate of p in the standard error for the test, but not for the interval

Always write the conclusion **in context**, describing each error and **its consequence in the problem's own terms** – that sentence is where the marks are.

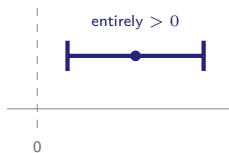


Comparing two proportions

A **two-sample z-interval** z :

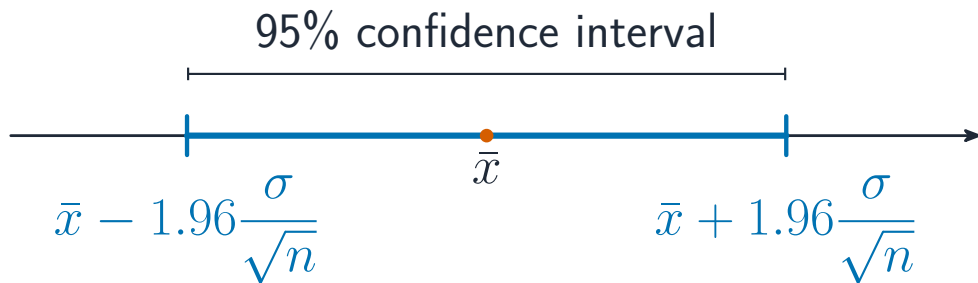
$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

If the interval excludes 0, the groups truly differ. The test pools: $\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$.



Excludes 0 $\Rightarrow p_1 > p_2$.

Confidence Interval for a Proportion, continued



A 95% confidence interval reaches 1.96 standard errors each side of the estimate

Why Be Normal?

Because a sample proportion $\hat{p}$ is approximately **normally** distributed (when the conditions hold).

This is what makes **inference** – drawing conclusions about a population from a sample – possible.

Exam tips

- State the conditions (random, 10%, large counts $np, nq \geq 10$) before any proportion inference.
- A **confidence interval** = estimate $\pm$ margin of error; “95% confident” refers to the **method’s** long-run capture rate.
- For a **test**, write H_0 and H_a , compute the test statistic, find the p-value, and compare to α .
- A small p-value is evidence **against** H_0 ; failing to reject does **not** prove H_0 .
- Larger samples shrink the margin of error; a higher confidence level widens it.



Unit 6 – key takeaways

1

Interval

$\hat{p} \pm z^* \cdot \text{SE}$; 95% of intervals catch p

2

Test

$H_0 : p = p_0$; z uses p_0 ; small p rejects

3

Errors

Type I (α) false alarm; Type II (β); power $1 - \beta$

4

Two groups

interval excluding 0 $\Rightarrow$ real difference; pool for the test

Check yourself

- 1 What is z^* for a 95% confidence interval?
- 2 A small p-value is evidence for or against H_0 ?
- 3 Rejecting a true H_0 is which type of error?
- 4 What value must a difference interval exclude to show a real gap?



Answers 1. 1.96 2. against H_0 3. Type I 4. zero