# 7.8 Setting Up a Test for the Difference of Two Population Means

Name: _____ Class: _____ Date: _____ **Total: 18 marks**

**KEY WORDS** two-sample $t$ test $t$ • matched pairs • standard error • conservative • twins • memory

## Objective

Build the skills to answer exam questions on **setting up a test for a difference of two means** .

**You must be able to:**

- write hypotheses about $\mu_1 - \mu_2$
- check the conditions for a two-sample $t$ test
- calculate the **two-sample $t$ statistic** $t$

## 1 Key knowledge and worked examples

### ■ The two-sample $t$ test

| idea | rule |
|---|---|
| hypotheses | $H_0 : \mu_1 - \mu_2 = 0$; $H_a : \mu_1 - \mu_2 > 0, < 0$ or $\neq 0$ |
| test statistic | $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$ |
| degrees of freedom | technology, or the smaller of $n_1 - 1$ and $n_2 - 1$ |
| conditions | Random (two independent samples, or random assignment); 10% for samples; Normal/Large Sample for each group |
| two-sample or paired | two separate groups of individuals: two-sample; two measurements on the same individuals or matched pairs: paired |

**Answer shapes:** define $\mu_1$ and $\mu_2$ in words and keep the order of subtraction throughout.



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## 2 Practice

**2.1** A teacher thinks that students who do homework online score higher on average than students who do it on paper. State the hypotheses. [2]

online homework mean $\mu_O$ paper homework mean $\mu_P$

**2.2** $\bar{x}_1 - \bar{x}_2 = 4$ and the standard error is 1.414. Calculate the test statistic. [2]



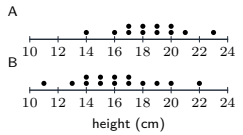

**2.3** Twenty pairs of twins: one twin uses a new toothpaste and the other the usual one. Explain whether a two-sample $t$ test is suitable. [2]




## 3 Exam-style questions

**3.1** Plants were randomly assigned to fertilizer A or B. A: $n = 12$, mean 18.4 cm, SD 2.6 cm. B: $n = 14$, mean 16.1 cm, SD 3.0 cm.

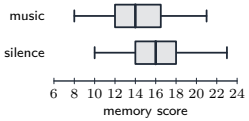
(a) State the hypotheses for a test of whether fertilizer A gives taller plants. [2]




(b) Calculate the test statistic. [2]

**3.2** In an experiment, 50 volunteers were randomly assigned to study a word list with music or in silence, then took a memory test. Music: $n = 25$, mean 14.2, SD 3.5. Silence: $n = 25$, mean 16.0, SD 3.1.

(a) State the hypotheses for a test of whether music changes the mean score. [2]




(b) Check the conditions. [2]

(c) Calculate the standard error and the test statistic. [2]

(d) State the degrees of freedom you would use without technology, and explain. [2]



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## Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

**2.1**

$H_0 : \mu_O - \mu_P = 0$, $H_a : \mu_O - \mu_P > 0$; $\mu_O$, $\mu_P$ = the true mean scores of students who do homework online and on paper

**2.2**

$t = 4/1.414$; $= 2.83$

**2.3**

not suitable; the twins form matched pairs, so a paired $t$ test on the differences should be used

**3.1**

(a) $H_0 : \mu_A - \mu_B = 0$, $H_a : \mu_A - \mu_B > 0$; $\mu_A$, $\mu_B$ = the true mean heights of plants like these with fertilizer A and B

(b) $SE = \sqrt{2.6^2/12 + 3.0^2/14} = 1.098$; $t = 2.3/1.098 = 2.09$

**3.2**

(a) $H_0 : \mu_M - \mu_S = 0$, $H_a : \mu_M - \mu_S \neq 0$; $\mu_M$, $\mu_S$ = the true mean scores of volunteers like these with music and in silence

(b) the volunteers were randomly assigned; both groups have $n = 25 < 30$, but the boxplots show no strong skew and no outliers

(c) $SE = \sqrt{3.5^2/25 + 3.1^2/25} = 0.935$; $t = -1.8/0.935 = -1.93$

(d) $df = 24$; the smaller of $n_1 - 1$ and $n_2 - 1$, which is a safe (conservative) choice



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