7.6 Confidence Intervals for the Difference of Two Means

Name: _____ Class: _____ Date: _____ **Total: 18 marks**

KEY WORDS two-sample t interval t • difference of means • conservative • independent groups • reaction time • random assignment

Objective

Build the skills to answer exam questions on **a confidence interval for the difference of two means** .

You must be able to:

- calculate the standard error of $\bar{x}_1 - \bar{x}_2$
- construct a two-sample t **interval** for $\mu_1 - \mu_2$
- interpret the interval, and say what the study design allows

1 Key knowledge and worked examples

■ Two independent groups

idea	rule
interval	$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
degrees of freedom	use technology, or the smaller of $n_1 - 1$ and $n_2 - 1$ (a safe, conservative choice)
conditions	Random: two independent random samples, or two groups from random assignment; 10% for samples; Normal/Large Sample for **each** group
not paired	use this only when the two groups are independent; paired data use the differences

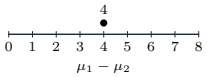
Answer shapes: say which df you used; with the conservative df, the interval is slightly wider than with technology.

2 Practice

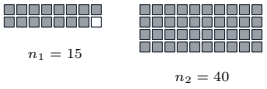
2.1 $s_1 = 5$, $n_1 = 25$, $s_2 = 6$ and $n_2 = 36$. Calculate the standard error of $\bar{x}_1 - \bar{x}_2$. [2]



2.2 Using Q2.1 with $\bar{x}_1 - \bar{x}_2 = 4$ and $df = 24$, construct a 95% confidence interval for $\mu_1 - \mu_2$. [2]



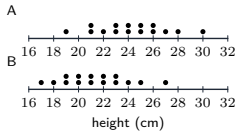
2.3 State the Normal/Large Sample condition for a two-sample t interval with $n_1 = 15$ and $n_2 = 40$. [2]



3 Exam-style questions

3.1 A gardener randomly assigns 30 seedlings to two soils, 15 to each. After 4 weeks, soil A gives heights with mean 24.3 cm and SD 3.1 cm; soil B, mean 21.6 cm and SD 2.8 cm.

(a) Calculate the standard error of $\bar{x}_A - \bar{x}_B$. [2]



(b) Construct a 95% confidence interval for $\mu_A - \mu_B$, using $df = 14$. [2]

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3.2 In an experiment, 40 volunteers were randomly assigned to drink coffee or water before a reaction test. Coffee: $n = 20$, mean 245 ms, SD 22 ms. Water: $n = 20$, mean 262 ms, SD 25 ms.

(a) Find $\bar{x}_C - \bar{x}_W$ and its standard error. [2]



(b) Construct a 95% confidence interval for $\mu_C - \mu_W$, using $df = 19$. [2]

(c) Interpret the interval in context. [2]

(d) Explain what the study design allows you to conclude. [2]

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Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

$\sqrt{25/25 + 36/36}$; $= 1.414$

2.2

$4 \pm 2.064(1.414) = 4 \pm 2.92$; (1.08, 6.92)

2.3

$n_2 = 40 \geq 30$ is enough for group 2; group 1 is small, so its data must show no strong skew and no outliers (or its population must be normal)

3.1

(a) $\sqrt{3.1^2/15 + 2.8^2/15}$; $= 1.08$ cm

(b) $2.7 \pm 2.145(1.08) = 2.7 \pm 2.31$; (0.39, 5.01) cm

3.2

(a) $245 - 262 = -17$ ms; $SE = \sqrt{22^2/20 + 25^2/20} = 7.45$ ms

(b) $-17 \pm 2.093(7.45) = -17 \pm 15.6$; $(-32.6, -1.4)$ ms

(c) we are 95% confident that the interval from -32.6 to -1.4 ms captures the true difference in mean reaction time (coffee minus water); so coffee lowers the mean reaction time by between about 1 and 33 ms

(d) the treatments were randomly assigned, so coffee causes the faster reactions; but only for volunteers like these, because they were not a random sample

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