

6.10 Setting Up a Test for the Difference of Two Population Proportions

Name: _____ Class: _____ Date: _____

Total: 8 marks

KEY WORDS pooled (combined) proportion • null hypothesis
• test statistic

Objective

Build the skills to answer exam questions on **setting up a test for a difference of two proportions**.

You must be able to:

- state the hypotheses $H_0 : p_1 = p_2$ and H_a
- explain why a **pooled (combined) proportion** is used
- write the pooled proportion $\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

■ Hypotheses

- **Null** $H_0 : p_1 = p_2$ (no difference).
- **Alternative** $H_a : p_1 \neq p_2$ (or one-sided).

■ The pooled proportion

Because H_0 assumes the proportions are **equal**, we combine both samples to estimate that common value:

$$\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2},$$

and use $\hat{p}_c$ in the standard error of the test statistic.

2 Practice

2.1 State the null hypothesis for a two-proportion test. [1]

2.2 State why a pooled proportion is used in the test. [1]

2.3 Write the formula for the pooled proportion. [1]

3 Exam-style questions

3.1 The null hypothesis in a two-proportion test is [1]

- ☐ **A** $p_1 \neq p_2$
B $p_1 = p_2$
C $p_1 = 0$
D $\hat{p}_1 = \hat{p}_2$

3.2 The pooled proportion combines [1]

- ☐ **A** only group 1
B both groups' successes and sample sizes
C the two means
D nothing

3.3 Group 1 has 30 successes in 100; group 2 has 20 in 100.

(a) Find $\hat{p}_1$. [1]

(b) Find $\hat{p}_2$. [1]

(c) Find the pooled proportion $\hat{p}_c$. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $H_0 : p_1 = p_2$

2.2

- H_0 assumes the proportions are equal, so both samples estimate the same common value

2.3

- $\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$

3.1 B

- $p_1 = p_2$.

3.2 B

- both groups' successes and sample sizes.

3.3

(a)

- 0.30

(b)

- 0.20

(c)

- $\hat{p}_c = \frac{30 + 20}{100 + 100} = 0.25$