5.6 Sampling Distributions for Differences in Sample Proportions

Name: _____ Class: _____ Date: _____ **Total: 8 marks**

KEY WORDS sampling distribution

Objective

Build the skills to answer exam questions on **the sampling distribution of a difference in sample proportions**.

You must be able to:

- use the mean $\mu_{\hat{p}_1-\hat{p}_2} = p_1 - p_2$
- use the standard deviation $\sqrt{\dfrac{p_1(1-p_1)}{n_1} + \dfrac{p_2(1-p_2)}{n_2}}$

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

■ Difference in proportions

To compare two groups, use $\hat{p}_1 - \hat{p}_2$:

$$\mu_{\hat{p}_1-\hat{p}_2} = p_1 - p_2, \qquad \sigma = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}.$$

Notice the two variances **add** under the square root (the samples are independent).

■ A worked value

Two machines have defect rates $p_1 = 0.10$ ($n_1 = 100$) and $p_2 = 0.06$ ($n_2 = 150$). Then

$$\mu = 0.10 - 0.06 = 0.04, \qquad \sigma = \sqrt{\tfrac{0.10(0.90)}{100} + \tfrac{0.06(0.94)}{150}} \approx \sqrt{0.001276} \approx 0.036.$$

2 Practice

2.1 Write the mean of the sampling distribution of $\hat{p}_1 - \hat{p}_2$. [1]

2.2 For $p_1 = 0.5$, $n_1 = 50$, $p_2 = 0.4$, $n_2 = 50$, find the standard deviation. [2]

2.3 State what this sampling distribution is used to compare. [1]

3 Exam-style questions

3.1 The mean of $\hat{p}_1 - \hat{p}_2$ is [1]

- ☐ **A** $p_1 \cdot p_2$
- **B** $p_1 - p_2$
- **C** $p_1 + p_2$
- **D** 0

3.2 The standard deviation of $\hat{p}_1 - \hat{p}_2$ combines the two variances by [1]

- ☐ **A** subtracting them
- **B** adding them
- **C** multiplying them
- **D** dividing them

3.3 $p_1 = 0.6$, $n_1 = 100$, $p_2 = 0.5$, $n_2 = 100$.

(a) Find the mean of $\hat{p}_1 - \hat{p}_2$. [1]

(b) Write the formula for its standard deviation. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $p_1 - p_2$

2.2

- $\sqrt{\frac{0.5(0.5)}{50} + \frac{0.4(0.6)}{50}} = \sqrt{0.005 + 0.0048} = \sqrt{0.0098} = 0.099$

2.3

- the difference between two population proportions

3.1 B

- $p_1 - p_2$.

3.2 B

- adding them.

3.3

(a)

- $0.6 - 0.5 = 0.1$

(b)

- $\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$