4.11 Parameters for a Binomial Distribution

Name: _____ Class: _____ Date: _____ **Total: 15 marks**

KEY WORDS random • probability • random variable • binomial

Objective

Build the skills to answer exam questions on **the parameters of a binomial distribution**.

You must be able to:

- calculate the **mean** of a binomial, $\mu_X = np$
- calculate the **standard deviation**, $\sigma_X = \sqrt{np(1-p)}$

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

■ Binomial parameters

For a binomial random variable with n trials and success probability p:

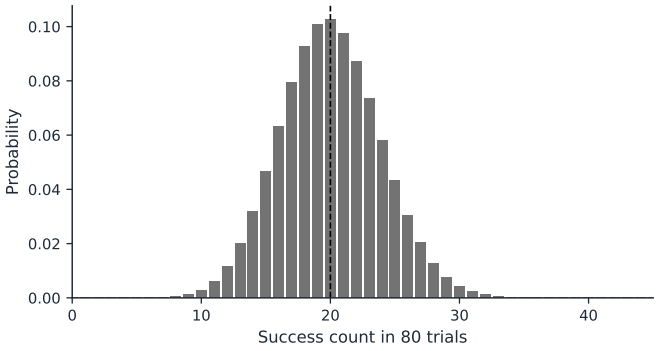
$$\mu_X = np, \qquad \sigma_X = \sqrt{np(1-p)}.$$

■ Example

$n = 100$, $p = 0.2$: $\mu_X = 100(0.2) = 20$ and $\sigma_X = \sqrt{100(0.2)(0.8)} = \sqrt{16} = 4$.

■ Worked application: Separate a typical count from a guaranteed count

Let X count successes in 80 independent trials, each with probability 0.25. The binomial assumptions include fixed trial number, two outcomes per trial and a constant success probability. Then $\mu = np = 20$ and $\sigma = \sqrt{np(1-p)} = \sqrt{15} \approx 3.87$ successes. A mean of 20 is the long-run average over repetitions, not a promise that this experiment produces exactly 20. The interval 20 ± 3.87 describes a scale of spread, not a guaranteed range. If trial outcomes influence one another, recalculate the model before using these formulas.



2 Practice

2.1 Write the formula for the mean of a binomial distribution. [1]

2.2 For $n = 50$, $p = 0.4$, find the mean. [2]

2.3 For $n = 100$, $p = 0.5$, find the standard deviation. [2]

3 Exam-style questions

3.1 The mean of a binomial distribution is [1]

- **A** n
- **B** np
- **C** p
- **D** $\sqrt{np}$

3.2 The standard deviation of a binomial distribution is [1]

- **A** np
- **B** $\sqrt{np(1-p)}$
- **C** $n(1-p)$
- **D** p

3.3 A binomial distribution has $n = 200$ and $p = 0.3$.

(a) Find the mean. [2]

(b) Find the standard deviation. [2]

3.4 Let X count successes in 120 independent trials with constant success probability 0.10. (a) Calculate its mean and standard deviation. [2]

(b) Interpret the mean in context and explain why changing success probability from trial to trial prevents this binomial model. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $\mu_X = np$

2.2

- $\mu_X = 50 \times 0.4 = 20$

2.3

- $\sigma_X = \sqrt{100(0.5)(0.5)} = \sqrt{25} = 5$

3.1 B

- np.

3.2 B

- $\sqrt{np(1-p)}$.

3.3

(a)

- $\mu_X = 200 \times 0.3 = 60$

(b)

- $\sigma_X = \sqrt{200(0.3)(0.7)} = \sqrt{42} \approx 6.5$

3.4

(a) $\mu = np = 12$; $\sigma = \sqrt{120(0.1)(0.9)} = \sqrt{10.8} \approx 3.29$.

(b) Repeated sets of 120 such trials average 12 successes; the binomial model requires the same success probability on each trial.