

Exercise walk-through

8.6

Carrying Out a Chi-Square Test for Homogeneity or Independence

AP Statistics

Homogeneity vs independence

Same χ^2 mechanics – the *design* sets the name:

Homogeneity

several separate samples, one variable

H_0 : same distribution across groups

Independence

one sample, two variables recorded

H_0 : the two variables are unrelated

Separately survey men and women = homogeneity; one sample recording gender & preference = independence.

Three chi-square tests, one calculation

the shared arithmetic

$\chi^2 = \sum \frac{(O - E)^2}{E}$ in every case – only the question and the df change

goodness-of-fit

one categorical variable against a claimed distribution. $df = k - 1$

test for homogeneity

several populations, one variable – are the distributions the same? Data from separate samples

test for independence

one population, two variables – are they associated? Data from one sample

the two-way df

$(\text{rows} - 1)(\text{columns} - 1)$ for both

the condition

every expected count at least 5

Homogeneity and independence use identical arithmetic and answer different questions. Which you have is decided by **how the data were collected**, not by the table.

The three chi-square tests

Goodness-of-fit

The **goodness-of-fit (GOF)** test: one categorical variable against a claimed distribution. Expected counts come from the claim.

Homogeneity

Several populations, one variable: are the distributions the same across the groups?

Independence

One population, two variables: are they associated? Expected counts are
$$\frac{\text{row total} \times \text{column total}}{n}.$$

Choosing

Count the samples and the variables. One sample, one variable is GOF; several samples is homogeneity; one sample, two variables is independence.

All three use the same statistic and the same conditions – expected counts at least 5. Only the degrees of freedom and the conclusion differ.

Exercises 2.1-2.3

2.1 State the steps to carry out a chi-square test for a two-way table. [2]

2.2 State what a small p-value indicates about the two variables. [1]

2.3 A test of independence gives a p-value of 0.02 at $\alpha = 0.05$. State the decision. [1]

Exercises 2.1-2.3

2.1 State the steps to carry out a chi-square test for a two-way table. [2]

2.1

- state hypotheses and check conditions; compute expected counts and χ^2 ; find the p-value; compare to α and conclude
-

2.2 State what a small p-value indicates about the two variables. [1]

2.2

- the two variables are associated (not independent)
-

2.3 A test of independence gives a p-value of 0.02 at $\alpha = 0.05$. State the decision. [1]

2.3

- reject H_0 ($0.02 < 0.05$)

Exercises 3.1-3.2

3.1 A chi-square test of independence with a small p-value suggests the variables are [1]

☐

A independent

B associated

C exactly equal

D unrelated

3.2 A chi-square test rejects H_0 when the p-value is

[1]

☐

A above α

B below α

C exactly zero

D exactly one

Exercises 3.1-3.2

3.1 A chi-square test of independence with a small p-value suggests the variables are [1]

☐

A independent

B associated

C exactly equal

D unrelated

3.1 B

- associated.

3.2 A chi-square test rejects H_0 when the p-value is

[1]

☐

A above α

B below α

C exactly zero

D exactly one

3.2 B

- below α .

Exercise 3.3 (a)

3.3 A test of independence gives $\chi^2 = 10$, $df = 2$, p-value 0.007 at $\alpha = 0.05$.

(a) State the decision.

[1]

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3.3 A test of independence gives $\chi^2 = 10$, $\text{df} = 2$, p-value 0.007 at $\alpha = 0.05$.

(a) State the decision.

[1]

(a)

- reject H_0

Exercise 3.3 (b)

3.3 A test of independence gives $\chi^2 = 10$, $df = 2$, p-value 0.007 at $\alpha = 0.05$.

(b) State whether the variables are associated.

[1]

Exercise 3.3 (b)

3.3 A test of independence gives $\chi^2 = 10$, $df = 2$, p-value 0.007 at $\alpha = 0.05$.

(b) State whether the variables are associated.

[1]

(b)

- yes, they are associated

Exercise 3.3 (c)

3.3 A test of independence gives $\chi^2 = 10$, $df = 2$, p-value 0.007 at $\alpha = 0.05$.

(c) State the conclusion in context.

[1]

Exercise 3.3 (c)

3.3 A test of independence gives $\chi^2 = 10$, $df = 2$, p-value 0.007 at $\alpha = 0.05$.

(c) State the conclusion in context.

[1]

(c)

- there is convincing evidence of an association between the two variables