

Exercise walk-through

6.8

Confidence Intervals for the Difference of Two Proportions

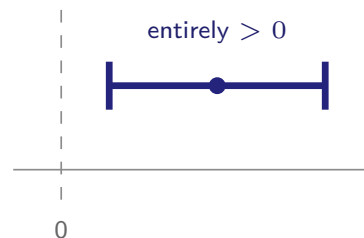
AP Statistics

Comparing two proportions

A two-sample z-interval z :

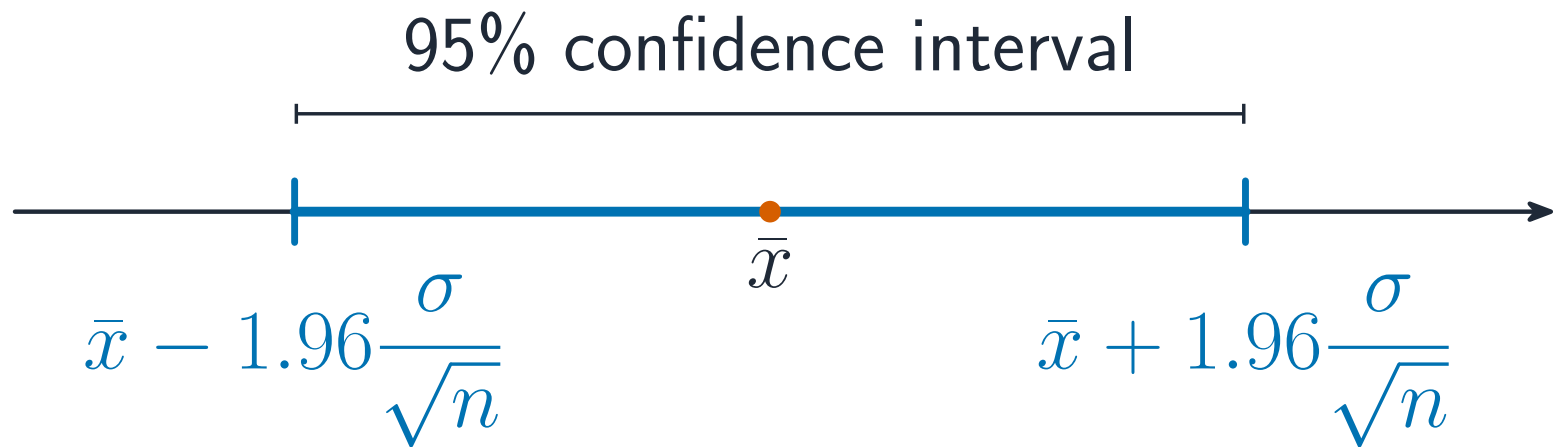
$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

If the interval excludes 0, the groups truly differ. The test pools: $\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$.



Excludes 0 $\Rightarrow p_1 > p_2$.

Confidence Interval for a Proportion, continued



A 95% confidence interval reaches 1.96 standard errors each side of the estimate

Why Be Normal?

Because a sample proportion $\hat{p}$ is approximately **normally** distributed (when the conditions hold).

This is what makes **inference** – drawing conclusions about a population from a sample – possible.

Exam tips

- State the conditions (random, 10%, large counts $np, nq \geq 10$) before any proportion inference.
- A **confidence interval** = estimate $\pm$ margin of error; “95% confident” refers to the **method’s** long-run capture rate.
- For a **test**, write H_0 and H_a , compute the test statistic, find the p-value, and compare to α .
- A small p-value is evidence **against** H_0 ; failing to reject does **not** prove H_0 .
- Larger samples shrink the margin of error; a higher confidence level widens it.

Exercises 2.1·2.3

2.1 Write the point estimate for the difference of two proportions. [1]

2.2 State how to judge whether the two proportions differ using the interval. [1]

2.3 For $\hat{p}_1 = 0.6$ and $\hat{p}_2 = 0.5$, find the point estimate of the difference. [1]

Exercises 2.1-2.3

2.1 Write the point estimate for the difference of two proportions. [1]

2.1

- $\hat{p}_1 - \hat{p}_2$

2.2 State how to judge whether the two proportions differ using the interval. [1]

2.2

- if the interval contains 0, there is no significant difference; if not, there is

2.3 For $\hat{p}_1 = 0.6$ and $\hat{p}_2 = 0.5$, find the point estimate of the difference. [1]

2.3

- $0.6 - 0.5 = 0.1$

Exercises 3.1-3.2

3.1 A confidence interval for $p_1 - p_2$ is centred on

[1]

☐

A $\hat{p}_1 \cdot \hat{p}_2$

B $\hat{p}_1 - \hat{p}_2$

C 0

D $\hat{p}_1 + \hat{p}_2$

3.2 If a 95% CI for $p_1 - p_2$ contains 0, the two proportions

[1]

☐

A definitely differ

B may not differ (no significant difference)

C are exactly equal

D are both zero

Exercises 3.1-3.2

3.1 A confidence interval for $p_1 - p_2$ is centred on

[1]

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A $\hat{p}_1 \cdot \hat{p}_2$

B $\hat{p}_1 - \hat{p}_2$

C 0

D $\hat{p}_1 + \hat{p}_2$

3.1 B

- $\hat{p}_1 - \hat{p}_2$.

3.2 If a 95% CI for $p_1 - p_2$ contains 0, the two proportions

[1]

☐

A definitely differ

B may not differ (no significant difference)

C are exactly equal

D are both zero

3.2 B

- may not differ (no significant difference).

Exercise 3.3 (a)

3.3 A 95% CI for $p_1 - p_2$ is $(0.02, 0.10)$.

(a) State which proportion is larger.

[1]

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3.3 A 95% CI for $p_1 - p_2$ is $(0.02, 0.10)$.

(a) State which proportion is larger.

[1]

(a)

- p_1 (the interval is positive)

Exercise 3.3 (b)

3.3 A 95% CI for $p_1 - p_2$ is $(0.02, 0.10)$.

(b) State whether 0 is in the interval.

[1]

Exercise 3.3 (b)

3.3 A 95% CI for $p_1 - p_2$ is $(0.02, 0.10)$.

(b) State whether 0 is in the interval.

[1]

(b)

- no

Exercise 3.3 (c)

3.3 A 95% CI for $p_1 - p_2$ is $(0.02, 0.10)$.

(c) State whether there is a significant difference.

[1]

Exercise 3.3 (c)

3.3 A 95% CI for $p_1 - p_2$ is $(0.02, 0.10)$.

(c) State whether there is a significant difference.

[1]

(c)

- yes