

Exercise walk-through

6.2

Constructing a Confidence Interval for a Population Proportion

AP Statistics

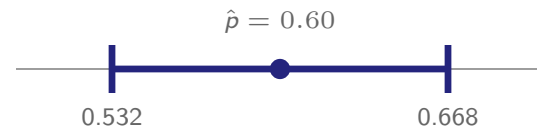
Confidence interval for a proportion

A **confidence interval** is estimate $\pm$ margin:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

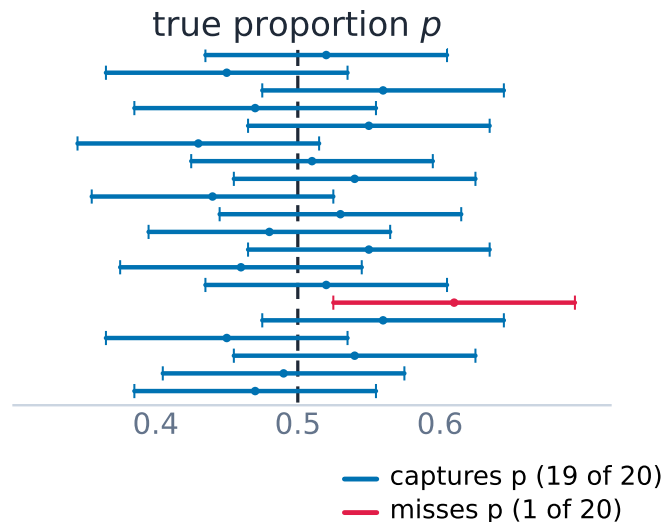
- **margin of error**
= $z^* \cdot \text{standard error}$
- for 95%: **critical value**
 $z^* = 1.96$

Check: random, $\leq 10\%$ of population,
and large counts.



Over many samples

95% CI: the method captures p in $\sim 95\%$ of samples (not a 95% chance that this one does)



Over many samples, about 95% of 95% confidence intervals capture the true proportion

Worked example – choosing the sample size

How many people must be surveyed for a 95% interval with margin of error at most 0.03?

- $z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \leq m$, solved for n
- use $\hat{p} = 0.5$ (the **worst case**, which maximises the product) and $z^* = 1.96$
- $n \geq \left(\frac{1.96}{0.03}\right)^2 (0.25) = 1067.1$
- round **up**: $n = 1068$

Two things are always marked: using $\hat{p} = 0.5$ when no estimate is given, and **rounding up** – rounding down would leave the margin too wide.

Exercises 2.1-2.3

2.1 Write the formula for a confidence interval for a proportion.

[1]

2.2 State the z^* value for a 95% confidence interval.

[1]

2.3 A poll gives $\hat{p} = 0.5$ with $n = 100$. Find the 95% margin of error.

[2]

Exercises 2.1-2.3

2.1 Write the formula for a confidence interval for a proportion.

[1]

2.1

- $$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

2.2 State the z^* value for a 95% confidence interval.

[1]

2.2

- 1.96

2.3 A poll gives $\hat{p} = 0.5$ with $n = 100$. Find the 95% margin of error.

[2]

2.3

- $$\text{ME} = 1.96 \sqrt{\frac{0.5(0.5)}{100}} = 1.96(0.05) = 0.098$$

Exercises 3.1-3.2

3.1 A confidence interval for p has the form

[1]

☐

A $\hat{p} \pm \text{ME}$

B $\hat{p} \times \text{ME}$

C $\hat{p}/\text{ME}$

D ME only

3.2 For a 95% confidence interval, z^* is about

[1]

☐

A 1.28

B 1.645

C 1.96

D 2.58

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- $\hat{p} \pm \text{ME}$.

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3.2 C

- 1.96.

Exercise 3.3 (a)

3.3 $\hat{p} = 0.4$, $n = 100$, 95% confidence.

(a) Find the margin of error.

[2]

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3.3 $\hat{p} = 0.4$, $n = 100$, 95% confidence.

(a) Find the margin of error.

[2]

(a)

- $$\text{ME} = 1.96 \sqrt{\frac{0.4(0.6)}{100}} = 1.96(0.049) = 0.096$$

Exercise 3.3 (b)

3.3 $\hat{p} = 0.4$, $n = 100$, 95% confidence.

(b) Give the confidence interval.

[1]

Exercise 3.3 (b)

3.3 $\hat{p} = 0.4$, $n = 100$, 95% confidence.

(b) Give the confidence interval.

[1]

(b)

- (0.304, 0.496)