

Exercise walk-through

6.1

Introducing Statistics: Why Be Normal?

AP Statistics

Setting up a test

A **significance test** weighs evidence against a claim.

■ **null** $H_0 : p = p_0$ (“no effect”)

■ **alternative** $H_a : p >, <, \text{ or } \neq p_0$

The **test statistic** (one-sample z-test), using p_0 :

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$$

Check conditions with the **assumed** p_0 , not $\hat{p}$.

A coin

58 heads in 100: $\hat{p} = 0.58$, $H_0 : p = 0.5$, $H_a : p > 0.5$.

$$z = \frac{0.58 - 0.50}{0.05} = 1.6$$

The two errors, and writing the conclusion

the significance level

α – fixed **before** the data are seen; usually 0.05

the alternative hypothesis

H_a – one-sided or two-sided, chosen from the question, never from the data

a Type I error

rejecting a **true** H_0 – a false alarm. Its probability is α

a Type II error

failing to reject a **false** H_0 – a missed detection. Its probability is β

power

$1 - \beta$: the chance of detecting a real effect. Raised by a larger sample or a larger α

two proportions

use the **combined (pooled)** estimate of p in the standard error for the *test*, but not for the interval

Always write the conclusion **in context**, describing each error and **its consequence in the problem's own terms** – that sentence is where the marks are.

Check yourself

- 1 What is z^* for a 95% confidence interval?
- 2 A small p-value is evidence for or against H_0 ?
- 3 Rejecting a true H_0 is which type of error?
- 4 What value must a difference interval exclude to show a real gap?



Answers 1. 1.96 2. against H_0 3. Type I 4. zero

Exercises 2.1-2.3

2.1 State why the normal distribution is central to inference for proportions. [1]

2.2 State the large counts condition for a proportion. [1]

2.3 State what inference uses a sample to estimate. [1]

Exercises 2.1-2.3

2.1 State why the normal distribution is central to inference for proportions. [1]

2.1

- the sampling distribution of $\hat{p}$ is approximately normal, allowing z -based inference
-

2.2 State the large counts condition for a proportion. [1]

2.2

- $np \geq 10$ and $n(1 - p) \geq 10$
-

2.3 State what inference uses a sample to estimate. [1]

2.3

- the unknown population proportion p

Exercises 3.1-3.2

3.1 Inference for proportions relies on the _____ distribution.

[1]

☐

A binomial

B normal

C uniform

D skewed

3.2 The large counts condition requires np and $n(1 - p)$ to each be at least

[1]

☐

A 1

B 5

C 10

D 30

Exercises 3.1-3.2

3.1 Inference for proportions relies on the _____ distribution.

[1]

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A binomial

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3.1 B

- normal.

3.2 The large counts condition requires np and $n(1 - p)$ to each be at least

[1]

☐

A 1

B 5

C 10

D 30

3.2 C

- 10.

Exercise 3.3 (a)

3.3 A sample proportion is used to estimate a population proportion.

(a) Name the distribution used for inference.

[1]

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(a) Name the distribution used for inference.

[1]

(a)

- the normal distribution

Exercise 3.3 (b)

3.3 A sample proportion is used to estimate a population proportion.

(b) State the condition for it to apply.

[1]

Exercise 3.3 (b)

3.3 A sample proportion is used to estimate a population proportion.

(b) State the condition for it to apply.

[1]

(b)

- $np \geq 10$ and $n(1 - p) \geq 10$

Exercise 3.3 (c)

3.3 A sample proportion is used to estimate a population proportion.

(c) State what we are estimating.

[1]

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[1]

(c)

- the population proportion p