

Exercise walk-through

# 5.6

## Sampling Distributions for Differences in Sample Proportions

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AP Statistics

# Sampling distribution of a proportion

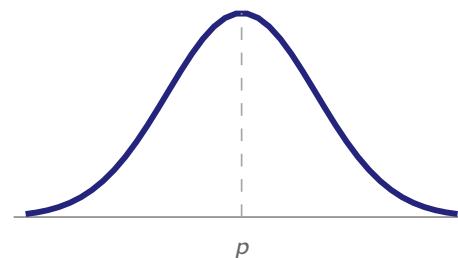
For a sample proportion  $\hat{p}$ :

$$\mu_{\hat{p}} = p \quad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

Normal when two conditions hold:

- **large counts** :  $np \geq 10$  and  $n(1-p) \geq 10$
- **10% condition** 10% :  $n \leq 0.10 N$

$\hat{p}$  is unbiased; bigger  $n$  under the root  
 $\Rightarrow$  less bounce.



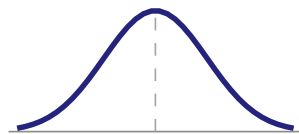
Centered on  $p$ ,  
spread  $\sigma_{\hat{p}}$ .

## Exam tips

- A **sampling distribution** is the distribution of a statistic over many samples, **centered on the true parameter**.
- The **Central Limit Theorem**: for a large enough sample the sample mean is approximately normal, even if the population is not.
- Larger samples give **less** variability (a smaller standard error).
- Check the conditions (random, independent/10%, large enough) before using a normal model.
- Keep straight what varies – the statistic – versus the fixed parameter.

# Check yourself

- 1 Is  $\mu$  a parameter or a statistic?
- 2 What does the CLT promise for large  $n$ ?
- 3 Write the standard error of a sample mean.
- 4 When combining independent variables, what adds?



**Answers** 1. parameter 2.  $\bar{x}$  is approximately normal 3.  $\sigma/\sqrt{n}$  4. the variances

## Exercises 2.1·2.3

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**2.1** Write the mean of the sampling distribution of  $\hat{p}_1 - \hat{p}_2$ . [1]

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**2.2** For  $p_1 = 0.5$ ,  $n_1 = 50$ ,  $p_2 = 0.4$ ,  $n_2 = 50$ , find the standard deviation. [2]

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**2.3** State what this sampling distribution is used to compare. [1]

## Exercises 2.1-2.3

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**2.1** Write the mean of the sampling distribution of  $\hat{p}_1 - \hat{p}_2$ . [1]

**2.1**

- $p_1 - p_2$
- 

**2.2** For  $p_1 = 0.5$ ,  $n_1 = 50$ ,  $p_2 = 0.4$ ,  $n_2 = 50$ , find the standard deviation. [2]

**2.2**

- $\sqrt{\frac{0.5(0.5)}{50} + \frac{0.4(0.6)}{50}} = \sqrt{0.005 + 0.0048} = \sqrt{0.0098} = 0.099$

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**2.3** State what this sampling distribution is used to compare. [1]

**2.3**

- the difference between two population proportions

## Exercises 3.1-3.2

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3.1 The mean of  $\hat{p}_1 - \hat{p}_2$  is

[1]

☐

A  $p_1 \cdot p_2$

B  $p_1 - p_2$

C  $p_1 + p_2$

D 0

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3.2 The standard deviation of  $\hat{p}_1 - \hat{p}_2$  combines the two variances by

[1]

☐

A subtracting them

B adding them

C multiplying them

D dividing them

## Exercises 3.1-3.2

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**3.1 B**

- $p_1 - p_2$ .

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**3.2** The standard deviation of  $\hat{p}_1 - \hat{p}_2$  combines the two variances by

[1]

☐

A subtracting them

B adding them

C multiplying them

D dividing them

**3.2 B**

- adding them.

## Exercise 3.3 (a)

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**3.3**  $p_1 = 0.6$ ,  $n_1 = 100$ ,  $p_2 = 0.5$ ,  $n_2 = 100$ .

(a) Find the mean of  $\hat{p}_1 - \hat{p}_2$ .

[1]

## Exercise 3.3 (a)

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**3.3**  $p_1 = 0.6$ ,  $n_1 = 100$ ,  $p_2 = 0.5$ ,  $n_2 = 100$ .

(a) Find the mean of  $\hat{p}_1 - \hat{p}_2$ .

[1]

(a)

- $0.6 - 0.5 = 0.1$

## Exercise 3.3 (b)

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**3.3**  $p_1 = 0.6$ ,  $n_1 = 100$ ,  $p_2 = 0.5$ ,  $n_2 = 100$ .

(b) Write the formula for its standard deviation.

[1]

## Exercise 3.3 (b)

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**3.3**  $p_1 = 0.6$ ,  $n_1 = 100$ ,  $p_2 = 0.5$ ,  $n_2 = 100$ .

(b) Write the formula for its standard deviation.

[1]

(b)

$$\bullet \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$