

Exercise walk-through

# 5.2

## The Normal Distribution, Revisited

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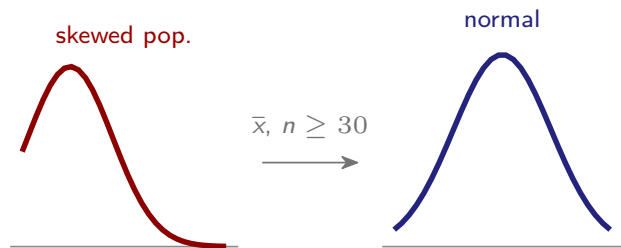
AP Statistics

# The Central Limit Theorem

The **CLT** : for large  $n$ , the sampling distribution of  $\bar{x}$  is **approximately normal** – *whatever* the population shape.

- AP guideline:  $n \geq 30$
- larger  $n \Rightarrow$  more normal & narrower

Averaging cancels the extremes, smoothing a skewed population into a bell.



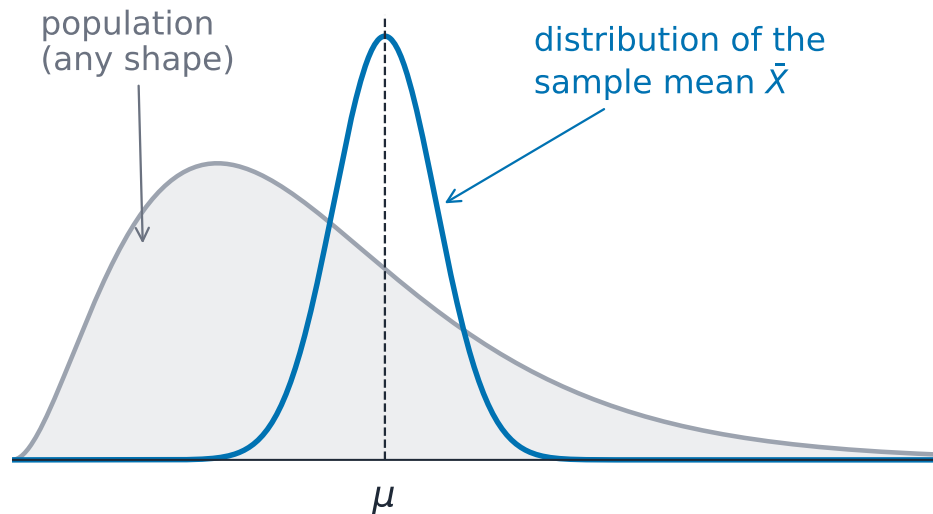
## Worked example – the standard error of a proportion

**40% of voters favour a measure ( $p = 0.4$ ) and you sample  $n = 100$ . Find the standard error, and the chance of seeing  $\hat{p} \geq 0.5$ .**

- $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.4(0.6)}{100}} = 0.049$
- check the conditions:  $np = 40$  and  $n(1-p) = 60$ , both  $\geq 10$ , so the distribution is approximately normal
- $z = \frac{0.5 - 0.4}{0.049} = 2.04$ , so  $P \approx 0.021$

The standard error shrinks as  $\sqrt{n}$ : to **halve** it you need **four times** the sample. That is why large surveys cost what they do.

# The Central Limit Theorem, continued



The sample mean is nearly normal whatever the shape of the population

## Exercises 2.1-2.3

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**2.1** Write the  $z$ -score formula.

[1]

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**2.2** For  $\mu = 60$ ,  $\sigma = 8$ , find the  $z$ -score of  $x = 76$ .

[2]

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**2.3** State what a  $z$ -score of 0 means.

[1]

## Exercises 2.1-2.3

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**2.1** Write the  $z$ -score formula.

[1]

**2.1**

- $z = \frac{x - \mu}{\sigma}$

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**2.2** For  $\mu = 60$ ,  $\sigma = 8$ , find the  $z$ -score of  $x = 76$ .

[2]

**2.2**

- $z = \frac{76 - 60}{8} = 2$

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**2.3** State what a  $z$ -score of 0 means.

[1]

**2.3**

- the value equals the mean

## Exercises 3.1-3.2

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**3.1** A  $z$ -score of 2 means a value is

[1]

☐

**A** at the mean

**B** 2 standard deviations above the mean

**C** 2 below the mean

**D** impossible

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**3.2** The standard normal distribution has mean and standard deviation

[1]

☐

**A** 0 and 1

**B** 1 and 0

**C** 100 and 15

**D** any values

## Exercises 3.1-3.2

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**3.1** A  $z$ -score of 2 means a value is

[1]

☐

A at the mean

B 2 standard deviations above the mean

C 2 below the mean

D impossible

**3.1 B**

- 2 standard deviations above the mean.

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**3.2** The standard normal distribution has mean and standard deviation

[1]

☐

A 0 and 1

B 1 and 0

C 100 and 15

D any values

**3.2 A**

- 0 and 1.

## Exercise 3.3 (a)

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**3.3** Test scores are normal with  $\mu = 500$  and  $\sigma = 100$ .

(a) Find the  $z$ -score for a score of 650.

[2]

## Exercise 3.3 (a)

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**3.3** Test scores are normal with  $\mu = 500$  and  $\sigma = 100$ .

(a) Find the  $z$ -score for a score of 650.

[2]

(a)

- $$z = \frac{650 - 500}{100} = 1.5$$

## Exercise 3.3 (b)

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**3.3** Test scores are normal with  $\mu = 500$  and  $\sigma = 100$ .

(b) State whether 650 is above or below average.

[1]

## Exercise 3.3 (b)

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**3.3** Test scores are normal with  $\mu = 500$  and  $\sigma = 100$ .

(b) State whether 650 is above or below average.

[1]

(b)

- above average