

Exercise walk-through

4.9

Combining Random Variables

AP Statistics

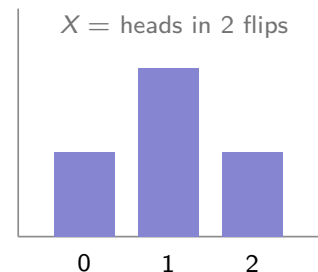
Random variables

A **random variable** X takes a number from a chance process (**discrete** or **continuous**).

- its **distribution** lists each x with $P(x)$, summing to 1

$$\mu_X = \sum x_i P(x_i) \quad \sigma_X^2 = \sum (x_i - \mu_X)^2 P(x_i)$$

μ_X is the **expected value** – the long-run average outcome.



Randomness, conditional probability, and expected value

random

individual outcomes are **uncertain**, but a regular pattern emerges over **many** repetitions

conditional probability

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

what it means

the chance of A **among the cases where B happened** – B becomes the new denominator

independence

A and B are independent when $P(A \mid B) = P(A)$

a probability distribution

lists every value of a random variable with its probability; they sum to 1

the mean (expected value)

$\mu_X = \sum x_i P(x_i)$ – the long-run average, not a value you expect to see

An expected value need not be attainable: the expected number of heads in three tosses is 1.5. It is an average over many repetitions.

Estimating Probabilities Using Simulation

A **simulation** imitates a chance process using random digits or technology.

Steps: state the model, assign digits to outcomes, run many trials, and record the proportion of trials meeting the condition.

Exercises 2.1-2.3

2.1 State the mean of $X + Y$ in terms of μ_X and μ_Y . [1]

2.2 If $\mu_X = 5$ and $\mu_Y = 3$, find the mean of $X + Y$. [1]

2.3 For $Y = 2X$ with $\sigma_X = 4$, find σ_Y . [2]

Exercises 2.1-2.3

2.1 State the mean of $X + Y$ in terms of μ_X and μ_Y . [1]

2.1

- $\mu_X + \mu_Y$
-

2.2 If $\mu_X = 5$ and $\mu_Y = 3$, find the mean of $X + Y$. [1]

2.2

- $5 + 3 = 8$
-

2.3 For $Y = 2X$ with $\sigma_X = 4$, find σ_Y . [2]

2.3

- $\sigma_Y = |2|\sigma_X = 2 \times 4 = 8$

Exercises 3.1-3.2

3.1 The mean of $X + Y$ is

[1]

☐

A $\mu_X \cdot \mu_Y$

B $\mu_X + \mu_Y$

C $\mu_X - \mu_Y$

D μ_X / μ_Y

☐

3.2 For **independent** X and Y , the variance of $X + Y$ is

[1]

A $\sigma_X + \sigma_Y$

B $\sigma_X^2 + \sigma_Y^2$

C $\sigma_X \cdot \sigma_Y$

D $\sigma_X^2 - \sigma_Y^2$

Exercises 3.1-3.2

3.1 The mean of $X + Y$ is

[1]

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A $\mu_X \cdot \mu_Y$

B $\mu_X + \mu_Y$

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3.1 B

- $\mu_X + \mu_Y$.

3.2 For **independent** X and Y , the variance of $X + Y$ is

[1]

☐

A $\sigma_X + \sigma_Y$

B $\sigma_X^2 + \sigma_Y^2$

C $\sigma_X \cdot \sigma_Y$

D $\sigma_X^2 - \sigma_Y^2$

3.2 B

- $\sigma_X^2 + \sigma_Y^2$.

Exercise 3.3 (a)

3.3 X has $\mu_X = 10$ and $\sigma_X = 2$. Let $Y = 3X$.

(a) Find μ_Y .

[1]

Exercise 3.3 (a)

3.3 X has $\mu_X = 10$ and $\sigma_X = 2$. Let $Y = 3X$.

(a) Find μ_Y .

[1]

(a)

- $\mu_Y = 3 \times 10 = 30$

Exercise 3.3 (b)

3.3 X has $\mu_X = 10$ and $\sigma_X = 2$. Let $Y = 3X$.

(b) Find σ_Y .

[2]

Exercise 3.3 (b)

3.3 X has $\mu_X = 10$ and $\sigma_X = 2$. Let $Y = 3X$.

(b) Find σ_Y .

[2]

(b)

- $\sigma_Y = |3| \times 2 = 6$