

Exercise walk-through

4.8

Mean and Standard Deviation of Random Variables

AP Statistics

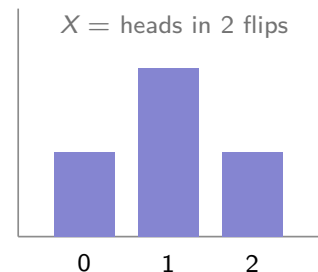
Random variables

A **random variable** X takes a number from a chance process (**discrete** or **continuous**).

- its **distribution** lists each x with $P(x)$, summing to 1

$$\mu_X = \sum x_i P(x_i) \quad \sigma_X^2 = \sum (x_i - \mu_X)^2 P(x_i)$$

μ_X is the **expected value** – the long-run average outcome.



Randomness, conditional probability, and expected value

random

individual outcomes are **uncertain**, but a regular pattern emerges over **many** repetitions

conditional probability

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

what it means

the chance of A **among the cases where B happened** – B becomes the new denominator

independence

A and B are independent when $P(A \mid B) = P(A)$

a probability distribution

lists every value of a random variable with its probability; they sum to 1

the mean (expected value)

$\mu_X = \sum x_i P(x_i)$ – the long-run average, not a value you expect to see

An expected value need not be attainable: the expected number of heads in three tosses is 1.5. It is an average over many repetitions.

Estimating Probabilities Using Simulation

A **simulation** imitates a chance process using random digits or technology.

Steps: state the model, assign digits to outcomes, run many trials, and record the proportion of trials meeting the condition.

Exercises 2.1-2.3

2.1 Write the formula for the mean of a discrete random variable. [1]

2.2 X takes 1 (prob 0.5) and 3 (prob 0.5). Find the mean. [2]

2.3 State what the mean of a random variable is also called. [1]

Exercises 2.1-2.3

2.1 Write the formula for the mean of a discrete random variable. [1]

2.1

- $\mu_X = \sum x_i \cdot P(x_i)$

2.2 X takes 1 (prob 0.5) and 3 (prob 0.5). Find the mean. [2]

2.2

- $\mu_X = 1(0.5) + 3(0.5) = 2$

2.3 State what the mean of a random variable is also called. [1]

2.3

- the expected value

Exercises 3.1-3.2

3.1 The expected value of X is

[1]

☐

A $\sum x_i$

B $\sum x_i \cdot P(x_i)$

C $\sum P(x_i)$

D the largest value of x

3.2 The mean of a random variable is also called its

[1]

☐

A median

B expected value

C mode

D variance

Exercises 3.1-3.2

3.1 The expected value of X is

[1]

☐

A $\sum x_i$

B $\sum x_i \cdot P(x_i)$

C $\sum P(x_i)$

D the largest value of x

3.1 B

- $\sum x_i \cdot P(x_i)$.

3.2 The mean of a random variable is also called its

[1]

☐

A median

B expected value

C mode

D variance

3.2 B

- expected value.

Exercise 3.3 (a)

3.3 X takes 2 (prob 0.4) and 5 (prob 0.6).

(a) Find the mean.

[2]

Exercise 3.3 (a)

3.3 X takes 2 (prob 0.4) and 5 (prob 0.6).

(a) Find the mean.

[2]

(a)

- $\mu_X = 2(0.4) + 5(0.6) = 0.8 + 3 = 3.8$

Exercise 3.3 (b)

3.3 X takes 2 (prob 0.4) and 5 (prob 0.6).

(b) State what the probabilities sum to.

[1]

Exercise 3.3 (b)

3.3 X takes 2 (prob 0.4) and 5 (prob 0.6).

(b) State what the probabilities sum to.

[1]

(b)

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