

Exercise walk-through

# 4.6

## Independent Events and Unions of Events

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AP Statistics

# Randomness, conditional probability, and expected value

**random**

individual outcomes are **uncertain**, but a regular pattern emerges over **many** repetitions

**conditional probability**

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

**what it means**

the chance of  $A$  **among the cases where  $B$  happened** –  $B$  becomes the new denominator

**independence**

$A$  and  $B$  are independent when  $P(A \mid B) = P(A)$

**a probability distribution**

lists every value of a random variable with its probability; they sum to 1

**the mean (expected value)**

$\mu_X = \sum x_i P(x_i)$  – the long-run average, not a value you expect to see

An expected value need not be attainable: the expected number of heads in three tosses is 1.5. It is an average over many repetitions.

# Combining random variables

A **linear transformation** :

$$\mu_{aX+b} = a\mu_X + b \quad \sigma_{aX+b} = |a| \sigma_X$$

For a **sum or difference** of **independent** variables, means combine and **variances add** – for *both*:

$$\mu_{X \pm Y} = \mu_X \pm \mu_Y \quad \sigma_{X \pm Y}^2 = \sigma_X^2 + \sigma_Y^2$$

Never subtract variances for a difference  
– variances always **add**.

## Two packages

$\sigma_X = 0.3$ ,  $\sigma_Y = 0.4$ , independent.

$$\sigma_{X+Y} = \sqrt{0.3^2 + 0.4^2} = 0.5$$

The difference  $X - Y$  has the *same*  $\sigma = 0.5$ .

# Exam tips

- A probability lies in  $[0, 1]$ ; use the **complement**  $(1 - P)$  and add mutually exclusive events.
- For independent events multiply; for “and/or” use the general addition and conditional rules.
- **Expected value**  $= \sum(\text{value} \times \text{probability})$ .
- Recognise **binomial** (fixed  $n$ , two outcomes, constant  $p$ ) and **geometric** settings.
- Draw a tree or table for multi-stage problems and multiply along branches.

## Exercises 2.1·2.3

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**2.1** State the condition for  $A$  and  $B$  to be independent. [1]

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**2.2** For independent events with  $P(A) = 0.5$  and  $P(B) = 0.4$ , find  $P(A \cap B)$ . [2]

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**2.3** Write the addition rule for  $P(A \cup B)$ . [1]

## Exercises 2.1-2.3

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**2.1** State the condition for  $A$  and  $B$  to be independent. [1]

**2.1**

- knowing that one has occurred does not change the probability of the other
- 

**2.2** For independent events with  $P(A) = 0.5$  and  $P(B) = 0.4$ , find  $P(A \cap B)$ . [2]

**2.2**

- $P(A \cap B) = 0.5 \times 0.4 = 0.2$
- 

**2.3** Write the addition rule for  $P(A \cup B)$ . [1]

**2.3**

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

## Exercises 3.1·3.2

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**3.1** For independent events,  $P(A \cap B) =$

[1]

☐

**A**  $P(A) + P(B)$

**B**  $P(A) \cdot P(B)$

**C** 0

**D**  $P(A \mid B)$

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**3.2** The addition rule is  $P(A \cup B) =$

[1]

☐

**A**  $P(A) + P(B)$

**B**  $P(A) + P(B) - P(A \cap B)$

**C**  $P(A) \cdot P(B)$

**D**  $P(A) - P(B)$

## Exercises 3.1-3.2

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**3.1 B**

- $P(A) \cdot P(B)$ .

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**3.2** The addition rule is  $P(A \cup B) =$

[1]

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**A**  $P(A) + P(B)$

**B**  $P(A) + P(B) - P(A \cap B)$

**C**  $P(A) \cdot P(B)$

**D**  $P(A) - P(B)$

**3.2 B**

- $P(A) + P(B) - P(A \cap B)$ .

## Exercise 3.3 (a)

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**3.3**  $P(A) = 0.6$ ,  $P(B) = 0.3$ , and  $P(A \cap B) = 0.2$ .

(a) Find  $P(A \cup B)$ .

[2]

## Exercise 3.3 (a)

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**3.3**  $P(A) = 0.6$ ,  $P(B) = 0.3$ , and  $P(A \cap B) = 0.2$ .

(a) Find  $P(A \cup B)$ .

[2]

(a)

- $0.6 + 0.3 - 0.2 = 0.7$

## Exercise 3.3 (b)

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**3.3**  $P(A) = 0.6$ ,  $P(B) = 0.3$ , and  $P(A \cap B) = 0.2$ .

(b) State whether  $A$  and  $B$  are independent (check  $P(A \cap B) = P(A) \cdot P(B)$ ). [1]

## Exercise 3.3 (b)

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**3.3**  $P(A) = 0.6$ ,  $P(B) = 0.3$ , and  $P(A \cap B) = 0.2$ .

(b) State whether  $A$  and  $B$  are independent (check  $P(A \cap B) = P(A) \cdot P(B)$ ). [1]

(b)

- no –  $P(A) \cdot P(B) = 0.18 \neq 0.2$