

Exercise walk-through

4.11

Parameters for a Binomial Distribution

AP Statistics

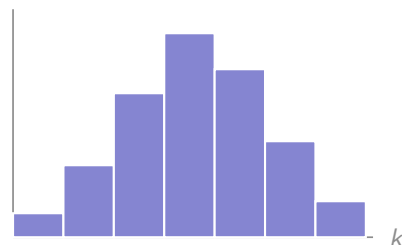
The binomial distribution

The **binomial setting** counts successes – remember **BINS**: **B**inary, **I**ndependent, fixed **N**umber, **S**ame p .

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

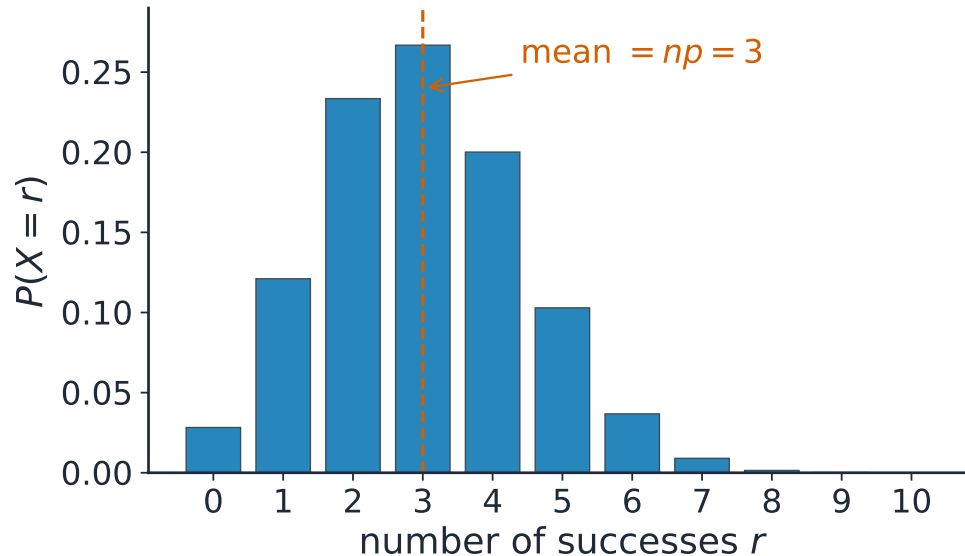
$$\mu_X = np \quad \sigma_X = \sqrt{np(1 - p)}$$

Shape: symmetric at $p = 0.5$, skewed otherwise; more bell-shaped as n grows.



Binomial: count of successes.

Introduction to the Binomial Distribution



The binomial distribution, with mean n times p

Worked example – binomial by hand

A free-throw shooter makes 80%. In $n = 10$ attempts, find $P(X = 8)$ and describe the distribution's centre and spread.

$$P(X = 8) = \binom{10}{8} (0.8)^8 (0.2)^2$$

$$= 45(0.1678)(0.04) \approx 0.302$$

$$\mu = np = 10(0.8) = 8 \text{ makes}$$

$$\sigma = \sqrt{10(0.8)(0.2)} \approx 1.26$$

$\binom{10}{8} = 45$ counts **which** 8 shots dropped – forget it and you find one single ordering (0.0067), not the event. Check **BINS** first: without independence it is not binomial at all.

Exercises 2.1-2.3

2.1 Write the formula for the mean of a binomial distribution.

[1]

2.2 For $n = 50$, $p = 0.4$, find the mean.

[2]

2.3 For $n = 100$, $p = 0.5$, find the standard deviation.

[2]

Exercises 2.1-2.3

2.1 Write the formula for the mean of a binomial distribution.

[1]

2.1

- $\mu_X = np$

2.2 For $n = 50$, $p = 0.4$, find the mean.

[2]

2.2

- $\mu_X = 50 \times 0.4 = 20$

2.3 For $n = 100$, $p = 0.5$, find the standard deviation.

[2]

2.3

- $\sigma_X = \sqrt{100(0.5)(0.5)} = \sqrt{25} = 5$

Exercises 3.1-3.2

3.1 The mean of a binomial distribution is

[1]

☐

A n

B np

C p

D $\sqrt{np}$

3.2 The standard deviation of a binomial distribution is

[1]

☐

A np

B $\sqrt{np(1-p)}$

C $n(1-p)$

D p

Exercises 3.1-3.2

3.1 The mean of a binomial distribution is

[1]

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A n

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3.1 B

- np .

3.2 The standard deviation of a binomial distribution is

[1]

☐

A np

B $\sqrt{np(1-p)}$

C $n(1-p)$

D p

3.2 B

- $\sqrt{np(1-p)}$.

Exercise 3.3 (a)

3.3 A binomial distribution has $n = 200$ and $p = 0.3$.

(a) Find the mean.

[2]

Exercise 3.3 (a)

3.3 A binomial distribution has $n = 200$ and $p = 0.3$.

(a) Find the mean.

[2]

(a)

- $\mu_X = 200 \times 0.3 = 60$

Exercise 3.3 (b)

3.3 A binomial distribution has $n = 200$ and $p = 0.3$.

(b) Find the standard deviation.

[2]

Exercise 3.3 (b)

3.3 A binomial distribution has $n = 200$ and $p = 0.3$.

(b) Find the standard deviation.

[2]

(b)

- $\sigma_X = \sqrt{200(0.3)(0.7)} = \sqrt{42} \approx 6.5$

Exercise 3.4

3.4 Let X count successes in 120 independent trials with constant success probability 0.10. (a) Calculate its mean and standard deviation. [2]

(b) Interpret the mean in context and explain why changing success probability from trial to trial prevents this binomial model. [2]

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(b) Interpret the mean in context and explain why changing success probability from trial to trial prevents this binomial model. [2]

3.4

(a) $\mu = np = 12$; $\sigma = \sqrt{120(0.1)(0.9)} = \sqrt{10.8} \approx 3.29$.

(b) Repeated sets of 120 such trials average 12 successes; the binomial model requires the same success probability on each trial.