

Exercise walk-through

4.1

Introducing Statistics: Random and Non-Random Patterns?

AP Statistics

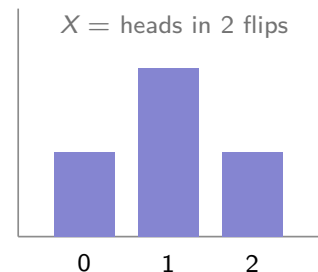
Random variables

A **random variable** X takes a number from a chance process (**discrete** or **continuous**).

- its **distribution** lists each x with $P(x)$, summing to 1

$$\mu_X = \sum x_i P(x_i) \quad \sigma_X^2 = \sum (x_i - \mu_X)^2 P(x_i)$$

μ_X is the **expected value** – the long-run average outcome.



Randomness, conditional probability, and expected value

random

individual outcomes are **uncertain**, but a regular pattern emerges over **many** repetitions

conditional probability

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

what it means

the chance of A **among the cases where B happened** – B becomes the new denominator

independence

A and B are independent when $P(A \mid B) = P(A)$

a probability distribution

lists every value of a random variable with its probability; they sum to 1

the mean (expected value)

$\mu_X = \sum x_i P(x_i)$ – the long-run average, not a value you expect to see

An expected value need not be attainable: the expected number of heads in three tosses is 1.5. It is an average over many repetitions.

Estimating Probabilities Using Simulation

A **simulation** imitates a chance process using random digits or technology.

Steps: state the model, assign digits to outcomes, run many trials, and record the proportion of trials meeting the condition.

Exercises 2.1-2.3

2.1 State what happens to a random process's behaviour in the long run. [1]

2.2 State the difference between random and non-random patterns. [1]

2.3 Give one example of a random process. [1]

Exercises 2.1-2.3

2.1 State what happens to a random process's behaviour in the long run. [1]

2.1

- it settles into a stable, predictable pattern (relative frequency)
-

2.2 State the difference between random and non-random patterns. [1]

2.2

- a random pattern is unpredictable in the short run; a non-random one is predictable
-

2.3 Give one example of a random process. [1]

2.3

- a coin flip, a die roll, or drawing a card (any one)

Exercises 3.1-3.2

3.1 A random process is unpredictable in the short run but shows a pattern

[1]

☐

A never

B in the long run

C only once

D immediately

3.2 Which of these is a random process?

[1]

☐

A the exact sunrise time tomorrow

B a coin flip

C a fixed timetable

D the value of $2 + 2$

Exercises 3.1-3.2

3.1 A random process is unpredictable in the short run but shows a pattern

[1]

☐

A never

B in the long run

C only once

D immediately

3.1 B

- in the long run.

3.2 Which of these is a random process?

[1]

☐

A the exact sunrise time tomorrow

B a coin flip

C a fixed timetable

D the value of $2 + 2$

3.2 B

- a coin flip.

Exercise 3.3 (a)

3.3 A fair coin is flipped many times.

(a) State whether the result of a single flip is predictable.

[1]

Exercise 3.3 (a)

3.3 A fair coin is flipped many times.

(a) State whether the result of a single flip is predictable.

[1]

(a)

- no

Exercise 3.3 (b)

3.3 A fair coin is flipped many times.

(b) State what emerges over many flips.

[1]

Exercise 3.3 (b)

3.3 A fair coin is flipped many times.

(b) State what emerges over many flips.

[1]

(b)

- a stable pattern – about half heads

Exercise 3.3 (c)

3.3 A fair coin is flipped many times.

(c) State the long-run proportion of heads for a fair coin.

[1]

Exercise 3.3 (c)

3.3 A fair coin is flipped many times.

(c) State the long-run proportion of heads for a fair coin.

[1]

(c)

- 0.5