Question 1: Function Concepts
Part A: Graphing calculator required — 6 points

The function *f* is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x.

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

Model Solution	Scoring	
A **(i)** The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation or indicate that it is not defined. Show the work that leads to your answer. **(ii)** Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.		
(i) $h(1) = g(f(1)) = g(1.75) = 0.333$	Value	**Point A1**
(ii) From the table, $f^{-1}(3.5) = 0$.	Value	**Point A2**

General Scoring Notes for Question 1 Parts A, B, and C

- Decimal approximations must be accurate to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 1 does not earn the point. For each additional part of Question 1 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

Scoring Notes for Part A

- **Point A1** is earned for a correct decimal approximation of 0.333 with supporting work of "$f(1)$" OR "$g(1.75)$" OR "1.75."
- **Point A2** does not require supporting work. **Point A2** is earned with a response of "0."

Partial Credit for Part A

A response that **does not** earn either **Point A1** or **Point A2** is eligible for **partial credit** in part A if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **0** for **Point A1** and **1** for **Point A2**.

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First Column	**Second Column**
Correct value in part A (i) without supporting work.	$f^{-1}(3.5)$ exists in part A (ii) without giving specific value.
Correct value in part A (i) that is not expressed as a decimal approximation.	
Correct value in part A (i) with a decimal presentation error.	

Model Solution	Scoring	
B **(i)** Find all values of x, as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values. **(ii)** Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.		
(i) $g(x) = 0 \Rightarrow -0.167x^3 + x^2 - 1.834 = 0$ $x = -1.233$, $x = 1.578$, $x = 5.643$	Values	**Point B1**
(ii) As x increases without bound, the output values of g decrease without bound. Therefore, $\lim_{x\to\infty} g(x) = -\infty$.	End behavior with limit notation	**Point B2**

Scoring Notes for Part B

- **Point B1** does not require supporting work. **Point B1** is earned with the three correct decimal approximations -1.233, 1.578, and 5.643. The use of "$x =$" is not required.
- **Point B2** requires a correct limit statement with four components: "lim", "$x \to \infty$", the function g, and $-\infty$. Examples that earn **Point B2** include:
  - $\lim_{x\to\infty} g(x) = -\infty$ OR $\lim_{x\to\infty} g = -\infty$
  - $\lim_{x\to\infty} g(x) \to -\infty$ OR $\lim_{x\to\infty} g \to -\infty$
  - $\lim_{x\to\infty} g(x) \quad -\infty$ OR $\lim_{x\to\infty} g \quad -\infty$
- If the response includes an additional, complete limit statement (e.g., $\lim_{x\to-\infty} g(x) = \infty$), the limit statement must be correct.

Partial Credit for Part B

A response that **does not** earn either **Point B1** or **Point B2** is eligible for **partial credit** in part B if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **1** for **Point B1** and **0** for **Point B2**.

First Column	Second Column
Three correct values in part B (i) with values that are not expressed as decimal approximations	Correct end behavior statement in part B (ii) without use of limit notation
Three correct values in part B (i) with a decimal presentation error	Correct end behavior statement in part B (ii) with incorrect limit notation
Only one correct value in part B (i) with no incorrect values included (may have a decimal presentation error)	Correct limit statement in part B (ii) that is missing "$x \to \infty$"
Only two correct values in part B (i) with no incorrect values included (may have a decimal presentation error)	

C **(i)** Based on the table, which of the following function types best models function *f*: linear, quadratic, exponential, or logarithmic?

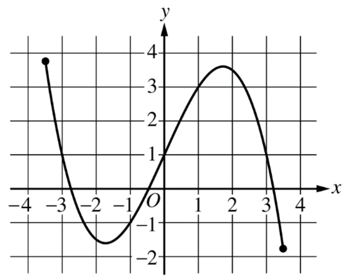
(ii) Give a reason for your answer in part C (i) based on the relationship between the change in the output values of *f* and the change in the input values of *f*. Refer to the values in the table in your reasoning.

(i) An exponential function best models *f*.	Answer	**Point C1**
(ii) In this case, the input-value intervals all have length 1. Examining the ratios of successive output values gives $\frac{f(-1)}{f(-2)} = \frac{f(0)}{f(-1)} = \frac{f(1)}{f(0)} = \frac{f(2)}{f(1)} = 0.5$. Because the successive output values over equal-length input-value intervals are proportional, an exponential model is best.	Reasoning	**Point C2**

Scoring Notes for Part C

- **Point C1** is earned for a correct function model with no incorrect function models listed. A response of "exponential" earns **Point C1**.
- Both **Point C1** and **Point C2** may be earned in part C (ii) provided there is no incorrect response in part C (i).
- **Point C2** requires an implicit or explicit reference to output values and input values from the table as support for the reason. For example, $\frac{f(n+1)}{f(n)} = 0.5$ OR "successive output values are decreasing proportionately by a factor of 0.5." The reasoning must demonstrate that the ratio of 0.5 applies to more than one pair of successive output values.
- A reason that references "exponential regression," "*r* values," OR "r^2 values" is not sufficient to earn **Point C2**.
- Special case: A response that indicates that *f* is best modeled by a **linear**, **quadratic**, or **logarithmic** function in part C (i) without a reason in part C (i) <u>combined</u> with a response in part C (ii) that provides both the correct answer <u>and</u> a correct reason is scored **0** for **Point C1** and **1** for **Point C2**.

Question 1: Function Concepts
Part A: Graphing calculator required — 6 points



Graph of f

The figure shows the graph of the function f on its domain of $-3.5 \le x \le 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of f. The function g is given by $g(x) = 2.916 \cdot (0.7)^x$.

Model Solution		Scoring
(A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined. (ii) Find all values of x for which $f(x) = 1$, or indicate that there are no such values.		
(i) $h(3) = g(f(3)) = g(1) = 2.041$	Value	**1 point**
(ii) From the graph, $f(x) = 1$ when $x = -3$, $x = 0$, and $x = 3$.	Values	**1 point**

General Scoring Notes for Question 1 Parts (A), (B), and (C):

- Decimal approximations must be correct to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 1 does not earn the point. For each additional part of Question 1 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

Scoring notes:

- The first point is earned for a correct decimal approximation of 2.041.
- The second point does not require supporting work.

- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **0-1** in Part (A).

First Column	**Second Column**
Correct value in (i) that is not expressed as a decimal approximation	Only one correct value in (ii) with no incorrect values included
Correct value in (i) with a decimal presentation error	Only two correct values in (ii) with no incorrect values included

(B) (i) Find all values of x, as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.

(ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

(i) $g(x) = 2 \Rightarrow 2.916(0.7)^x = 2$ $x = 1.057$	Value	**1 point**
(ii) As x increases without bound, the output values of g get arbitrarily close to 0. Therefore, $\lim_{x\to\infty} g(x) = 0$.	End behavior with limit notation	**1 point**

Scoring notes:

- The first point is earned for a correct decimal approximation of 1.057. No incorrect values may be included.
- The second point requires a correct limit statement with four components: "lim," "$x \to \infty$," the function g, and 0. Examples that earn the point include:
  - $\lim_{x\to\infty} g(x) = 0$ OR $\lim_{x\to\infty} g = 0$
  - $\lim_{x\to\infty} g(x) \to 0$ OR $\lim_{x\to\infty} g \to 0$
  - $\lim_{x\to\infty} g(x) \quad 0$ OR $\lim_{x\to\infty} g \quad 0$

  If the response includes an additional, complete limit statement (e.g., $\lim_{x\to-\infty} g(x) = \infty$), the value of the limit must be correct.

- A response that does not earn either point in Part (B) is eligible for **partial credit** in Part (B) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (B).

First Column	**Second Column**
Correct answer in (i) that is not expressed as a decimal approximation	Correct end behavior statement in (ii) without use of limit notation
Correct value in (i) with a decimal presentation error	Correct end behavior statement in (ii) with incorrect limit notation
	Correct limit statement in (ii) that is missing "$x \to \infty$"

(C) (i) Determine if f has an inverse function.

(ii) Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

(i) f does not have an inverse function on its domain of $-3.5 \le x \le 3.5$.	Answer	**1 point**
(ii) There are output values of f that are not mapped from unique input values; for example, $f(-3) = f(0) = f(3) = 1$.	Reason	**1 point**

Scoring notes:

- The first point is earned for a correct answer.
- Both points may be earned in (ii) provided there is no incorrect response in (i).
- The second point requires an implicit or explicit reference to the definition of a function AND support for the reason by referencing specific function values.
- A response such as "f does not have an inverse function because $f(-3) = f(0) = 1$" OR "f does not have an inverse function because there are two input values mapped to 1" earns both points.
- A response such as "f is not one-to-one" OR "f fails the horizontal line test" OR "There are output values that are not mapped from unique input values" is not sufficient to earn the second point.
- The second point cannot be earned if there are any errors in Part (C) (ii).
- A response that indicates that f **has** an inverse function in Part (C) (i) without a reason in Part (C) (i) combined with a response in Part (C) (ii) that provides both the correct answer and a correct reason is scored **0-1**.

Total for question 1 **6 points**

Question 4: Symbolic Manipulations
Part B: Graphing calculator not allowed — 6 points

Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

Model Solution	Scoring	

(A) The functions g and h are given by

$$g(x) = e^{(x+3)}$$

$$h(x) = \arcsin\left(\frac{x}{2}\right).$$

(i) Solve $g(x) = 10$ for values of x in the domain of g.

(ii) Solve $h(x) = \frac{\pi}{4}$ for values of x in the domain of h.

Model Solution	Scoring	
(i) $g(x) = 10$ $e^{(x+3)} = 10$ $\ln e^{(x+3)} = \ln 10$ $x + 3 = \ln 10$ $x = -3 + \ln 10$	Solution to $g(x) = 10$	**1 point**
(ii) $h(x) = \frac{\pi}{4}$ $\arcsin\left(\frac{x}{2}\right) = \frac{\pi}{4}$ $\frac{x}{2} = \sin\left(\frac{\pi}{4}\right)$ $\frac{x}{2} = \frac{\sqrt{2}}{2}$ (OR $\frac{1}{\sqrt{2}}$) $x = \sqrt{2}$	Solution to $h(x) = \frac{\pi}{4}$	**1 point**

Scoring notes:

- Supporting work is required in (i) and (ii). “Scratchwork” can be ignored; the use of a variable other than x is acceptable. Arithmetic errors following a correct solution may be considered scratchwork.
- Supporting work in (ii) must include $\frac{x}{2} = \sin\left(\frac{\pi}{4}\right)$ OR $\frac{x}{2} = \frac{\sqrt{2}}{2}$ OR $\frac{x}{2} = \frac{1}{\sqrt{2}}$.
- An alternate solution for (i) is $x = \frac{\log_b 10}{\log_b e} - 3$, where $b > 0$, $b \neq 1$, and the result is evaluated according to bullets 2 and 3 in the directions.
- Where applicable, answers that have not been evaluated according to bullets 2 and 3 in the directions do not earn the point. Rationalizing denominators is not required.
- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (A).

First Column	**Second Column**
Correct answer in (i) without supporting work	Correct answer in (ii) without supporting work
Correct answer in (i) with supporting work, but the answer has not been evaluated according to bullets 2 and 3 in the directions. No incorrect work.	Correct answer in (ii) with supporting work, but the answer has not been evaluated according to bullets 2 and 3 in the directions. No incorrect work.
Answer in (i) is reported as $x + 3 = \ln 10$. No incorrect work follows.	Answer in (ii) is reported as $\frac{x}{2} = \sin\left(\frac{\pi}{4}\right)$. No incorrect work follows.
	Answer in (ii) is reported as $\frac{x}{2} = \frac{\sqrt{2}}{2}$. No incorrect work follows.
	Answer in (ii) is reported as $\frac{x}{2} = \frac{1}{\sqrt{2}}$. No incorrect work follows.

(B) The functions j and k are given by

$$j(x) = \log_{10}\left(8x^5\right) + \log_{10}\left(2x^2\right) - 9\log_{10} x$$

$$k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right)\sec x .$$

(i) Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log_{10}(\text{expression})$.

(ii) Rewrite $k(x)$ as a single term involving $\tan x$.

(i) $j(x) = \log_{10}\left(8x^5\right) + \log_{10}\left(2x^2\right) - 9\log_{10} x$ $j(x) = \log_{10}\left(8x^5 \cdot 2x^2\right) - \log_{10} x^9$ $j(x) = \log_{10}\left(\frac{8x^5 \cdot 2x^2}{x^9}\right)$ $j(x) = \log_{10}\left(\frac{16x^7}{x^9}\right)$ $j(x) = \log_{10}\left(\frac{16}{x^2}\right),\ x > 0$	Expression for $j(x)$	**1 point**
(ii) $k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right)\sec x$ $k(x) = \left(\frac{\cos^2 x}{\sin x}\right)\left(\frac{1}{\cos x}\right)$ $k(x) = \left(\frac{\cos x}{\sin x}\right)$ $k(x) = \frac{1}{\tan x},\ \sin x \neq 0,\ \cos x \neq 0$	Expression for $k(x)$	**1 point**

Scoring notes:

- Supporting work is required in (i) and (ii). "Scratchwork" can be ignored; the use of a variable other than x is acceptable.
- Domain restrictions are not required to be included and are not scored.
- Where applicable, answers that have not been evaluated according to bullets 2 and 3 in the directions do not earn the point.
- To earn the first point, use of "log" rather than "$\log_{10}$" is acceptable.
- The expression $j(x) = \log_{10}\left(\frac{4}{x}\right)^2$ earns the point in (i) with supporting work.
- A logarithmic expression that is missing one or both parentheses around the full argument of the logarithm is still eligible to earn the point.
- If a response is presented as a complex fraction, the complex fraction must be unambiguous in structure. Parentheses must be used correctly, and/or the fraction bars must be clearly and correctly proportioned.

- A response that does not earn either point in Part (B) is eligible for **partial credit** in Part (B) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (B).

First Column	**Second Column**
Correct expression in (i) without supporting work	Correct expression in (ii) without supporting work
Expression in (i) is reported as $\log_{10}\left(\frac{8x^5 \cdot 2x^2}{x^9}\right)$. No incorrect work follows.	Expression in (ii) is reported as $\frac{\cos x}{\sin x}$ OR $\cot x$. No incorrect work follows.
Expression in (i) is reported as $\log_{10}\left(\frac{16x^7}{x^9}\right)$. No incorrect work follows.	Expression in (ii) includes a correct application of a Pythagorean identity with no incorrect work.
Expression in (i) is reported as $2\log_{10}\left(\frac{4}{x}\right)$. No incorrect work follows.	
Expression in (i) is reported as $-\log_{10}\left(\frac{x^2}{16}\right)$. No incorrect work follows.	
Expression in (i) is reported as $-2\log_{10}\left(\frac{x}{4}\right)$. No incorrect work follows.	
The expression in (i) is reported using natural logarithm and has the correct argument OR any of the expressions in partial credit rows two through six above are presented with natural logarithm.	

(C) The function m is given by

$$m(x) = \cos^{-1}(\tan(2x)).$$

Find all values in the domain of m that yield an output value of 0.

$m(x) = 0 \Rightarrow \cos^{-1}(\tan(2x)) = 0$ $\tan(2x) = \cos(0)$	One value of x	**1 point**
$\tan(2x) = 1$ $2x = \frac{\pi}{4} + \pi n$ $x = \frac{\pi}{8} + \frac{\pi}{2}n$, where n is any integer	All values of x	**1 point**

Scoring notes:

- Supporting work is required. "Scratchwork" can be ignored; the use of a variable other than x is acceptable.
- A response with supporting work that gives all correct values for x, such as $x = \frac{\pi}{8} + \pi n$ and $x = \frac{5\pi}{8} + \pi n$, earns both points.
- When expressing a general solution for all values for x (e.g., $x = \frac{\pi}{8} + \frac{\pi}{2}n$), the response can use i, k, n, or any letter except x, which is the variable used in the function.
- To earn the second point, "where n is any integer" is not required to be included.
- To earn the second point, no incorrect values for x are included.

Total for question 4 **6 points**