

Parametric, Vectors & Matrices

AP Precalculus • Unit 4

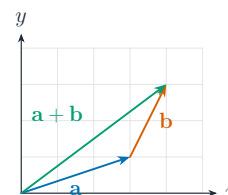
AP Precalculus



Parametric, Vectors & Matrices

What we will cover

- 1 Parametric functions
- 2 Planar motion & rates
- 3 Circles lines & conics
- 4 Implicit curves
- 5 Vectors
- 6 Matrices & determinants
- 7 Linear transformations



1 Parametric

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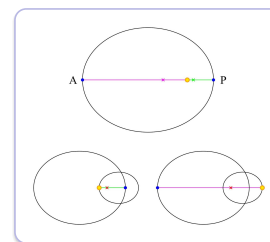
↳ Parametric

Parametric functions

A **parametric function** 参数函数 gives x and y each as functions of a **parameter** 参数 t :

$$x = x(t), \quad y = y(t).$$

The curve traced **need not be a function of x** – it may loop or cross itself.



Elliptical orbit

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↳ Parametric

Planar motion and rates of change

As t advances, $(x(t), y(t))$ traces the path of a moving point – so t carries **direction** and **timing**, which a plain curve cannot show.

Read each rate **separately**: $x(t)$ increasing moves right, $y(t)$ decreasing moves down.



Vector coaster

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↳ Parametric

Implicit curves and conic sections

An **implicitly defined** 隐式定义 curve is an equation in x and y – not solved for y . The **conic sections** 圆锥曲线:

- circle $x^2 + y^2 = r^2$
- ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
- parabola $y = ax^2$

To **parametrize** an implicit curve, find $x(t)$ and $y(t)$ that satisfy the equation for every t – for a circle, $\cos^2 t + \sin^2 t = 1$ does it **automatically**.

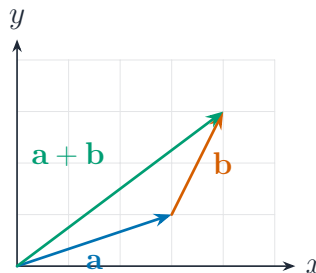
2 Vectors

Vectors

A **vector** 向量 has **magnitude** and **direction**, written $\langle a, b \rangle$:

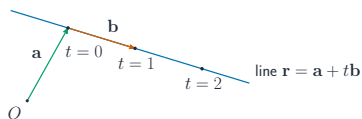
$$|\vec{v}| = \sqrt{a^2 + b^2}, \quad \theta = \arctan \frac{b}{a}.$$

Add **component-wise**; a **scalar** 标量 multiple $k\vec{v}$ scales the length (and flips it if $k < 0$).



Vector-valued functions

A **vector-valued function** 向量值函数 $\vec{p}(t) = \langle x(t), y(t) \rangle$ is a **position vector** for each t – the same idea as a parametric function, written as one object.



A line: start at $\vec{a}$, then slide t lots of the direction $\vec{b}$ – $\vec{p}(t) = \vec{a} + t\vec{b}$.

3 Matrices

Matrices, determinant and inverse

For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the **determinant** 行列式 is $\det A = ad - bc$.

A is **invertible** 可逆 exactly when $\det A \neq 0$. The inverse **undoes** the transformation.

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

$\det A = (3)(4) - (1)(2) = 10 \neq 0$, so A is invertible:

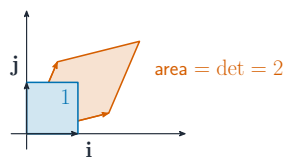
$$A^{-1} = \frac{1}{10} \begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix}.$$

Swap the diagonal, negate the off-diagonal, divide by $\det$.

Matrices as linear transformations

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \text{ stretches the plane}$$

It sends $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} 2 \\ 3 \end{bmatrix}$: $2\times$ horizontally, $3\times$ vertically. Its determinant $2 \times 3 = 6$ means **every area is multiplied by 6** – the unit square becomes a 2×3 rectangle.

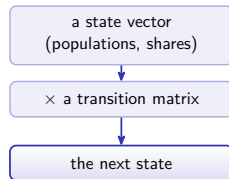


The determinant is an **area scale factor** – a negative one flips orientation.

Matrices as functions, and as models

A matrix is a function on vectors: composition of transformations is matrix multiplication, and the identity matrix leaves every vector unchanged.

Order matters: AB and BA usually differ – just like function composition.



4

Recap

Unit 4 – key takeaways

1 Parametric

t adds direction and timing a plain curve cannot show

2 Conics

implicit curves; parametrize with $\cos^2 t + \sin^2 t = 1$

3 Vectors

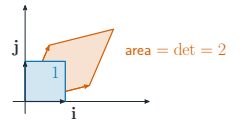
magnitude and direction; add component-wise

4 Matrices

$\det = ad - bc$ is the area scale factor; $\neq 0$ means invertible

Check yourself

- 1 Parametrize a circle of radius r centred at (h, k) .
- 2 Find $\det \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$.
- 3 What does a determinant of 6 do to areas?
- 4 Give the magnitude of $\langle a, b \rangle$.



Answers 1. $x = h + r \cos t$, $y = k + r \sin t$ 2. 10 3. multiplies every area by 6
4. $\sqrt{a^2 + b^2}$