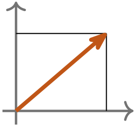
Parametric, Vectors & Matrices

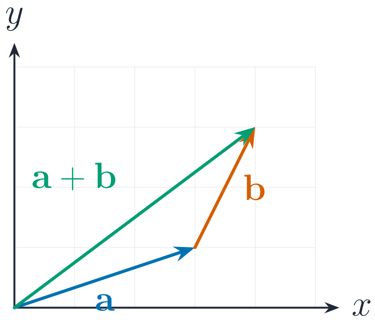
AP Precalculus ▪ Unit 4

AP Precalculus



What we will cover

1. Parametric functions
2. Planar motion & rates
3. Circles lines & conics
4. Implicit curves
5. Vectors
6. Matrices & determinants
7. Linear transformations

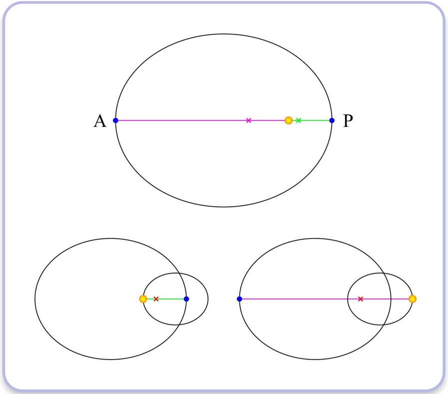


Parametric functions

A **parametric function** 参数函数 gives x and y each as functions of a **parameter** 参数 t:

$$x = x(t), \qquad y = y(t).$$

> The curve traced need not be a function of x – it may loop or cross itself.



elliptical Orbit

Planar motion and rates of change

As t advances, $(x(t), y(t))$ traces the path of a moving point – so t carries direction and timing, which a plain curve cannot show.

> Read each rate separately: $x(t)$ increasing moves right, $y(t)$ decreasing moves down.

vector Coaster

Implicit curves and conic sections

An **implicitly defined** 隐式定义 curve is an equation in x and y – not solved for y. The **conic sections** 圆锥曲线:

- circle $x^2 + y^2 = r^2$
- ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
- parabola $y = ax^2$

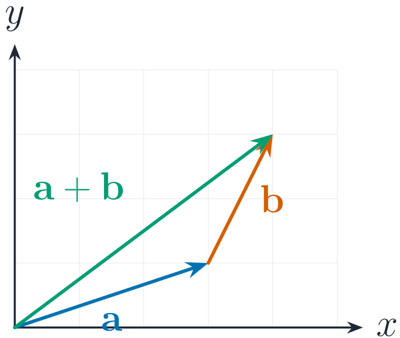
> To parametrize an implicit curve, find $x(t)$ and $y(t)$ that satisfy the equation for every t – for a circle, $\cos^2 t + \sin^2 t = 1$ does it automatically.

Vectors

A **vector** 向量 has magnitude and direction, written $\langle a, b\rangle$:

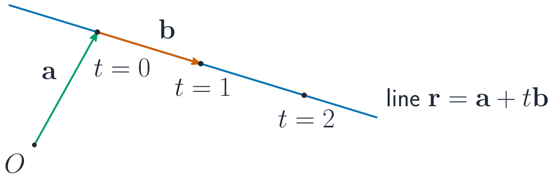
$$|\vec{v}| = \sqrt{a^2 + b^2}, \qquad \theta = \arctan\frac{b}{a}.$$

> Add component-wise; a **scalar** 标量 multiple $k\vec{v}$ scales the length (and flips it if $k < 0$).



Vector-valued functions

A **vector-valued function** 向量值函数 $\vec{p}(t) = \langle x(t), y(t) \rangle$ is a position vector for each t – the same idea as a parametric function, written as one object.



> A line: start at $\vec{a}$, then slide t lots of the direction $\vec{b}$ – $\vec{p}(t) = \vec{a} + t\vec{b}$.

Matrices, determinant and inverse

For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the **determinant** 行列式 is

$$\det A = ad - bc.$$

> A is **invertible** 可逆 exactly when $\det A \neq 0$. The inverse undoes the transformation.

$A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

$\det A = (3)(4) - (1)(2) = 10 \neq 0$, so A is invertible:

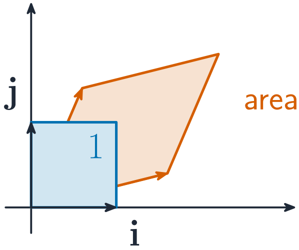
$$A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix}.$$

Swap the diagonal, negate the off-diagonal, divide by $\det$.

Matrices as linear transformations

$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ stretches the plane

It sends $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} 2 \\ 3 \end{bmatrix}$: $2\times$ horizontally, $3\times$ vertically. Its determinant $2 \times 3 = 6$ means every area is multiplied by 6 – the unit square becomes a 2×3 rectangle.



The determinant is an area scale factor – a negative one flips orientation.

Matrices as functions, and as models

A matrix is a function on vectors: composition of transformations is matrix multiplication, and the identity matrix leaves every vector unchanged.

> Order matters: AB and BA usually differ – just like function composition.

a state vector (populations, shares)

↓

$\times$ a transition matrix

↓

the next state

Unit 4 – key takeaways

1 Parametric

t adds direction and timing a plain curve cannot show

2 Conics

implicit curves; parametrize with $\cos^2 t + \sin^2 t = 1$

3 Vectors

magnitude and direction; add component-wise

4 Matrices

$\det = ad - bc$ is the area scale factor; $\neq 0$ means invertible

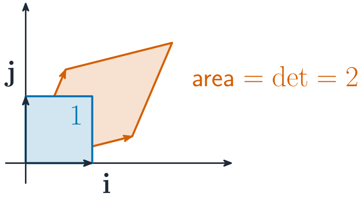
Check yourself

1. Parametrize a circle of radius r centred at (h, k).

2. Find $\det \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$.

3. What does a determinant of 6 do to areas?

4. Give the magnitude of $\langle a, b \rangle$.



Answers **1.** $x = h + r\cos t,\ \ y = k + r\sin t$ **2.** 10 **3.** multiplies every area by 6 **4.** $\sqrt{a^2 + b^2}$